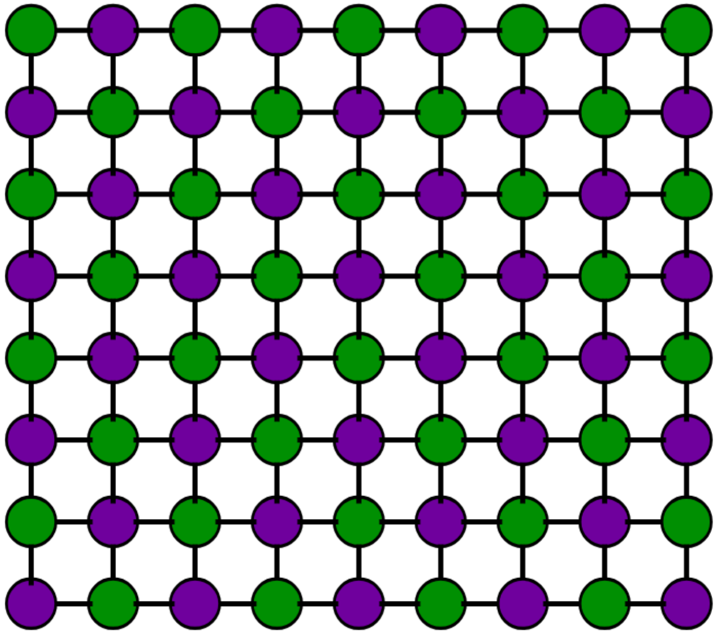
Thermodynamic Quantum Time-Space Crystals.

Konstantin B. Efetov

Ruhr University Bochum, Germany

Space Crystal



What is thermodynamic time-space crystal?

Long-range order in both space and time.

Existence of an order parameter oscillating in both space and time.

Konstantin B. Efetov   arXiv:1902.07520   arXiv:1905.04128

General expected properties (in agreement with laws of nature!).

Basic properties of Space Crystal (CDW).

Order parameter: $\rho(r-r_0)=\rho_0\cos(Q(r-r_0))$

r_0 is arbitrary

$$\int \rho(r-r_0)dr_0=0$$

$$K(r)_{r\to\infty}=\int \rho(r_0)\rho(r-r_0)dV_0 \propto \cos(Qr)$$

$\Updownarrow$

Existence of long range order in space.

R=(t,r)

Basic properties of time crystal.

Order parameter: $B(t-t_0)$ (Periodic in time).

t_0 is arbitrary in thermodynamic equilibrium!

$$\int B(t-t_0)dt_0=0$$

No dependence on time!

$$L(R-R_0)_{r\to\infty,t\to\infty}=\int B(-R_0)B(R-R_0)dR_0$$
$$\propto \cos(Qr)\cos(\Gamma t)$$

$\Updownarrow$

Existence of long range order in time-space!

Is such a behavior possible?

Previous “Time Crystals”……

F. Wilczek, Quantum Time Crystals, PRL, 109, 160401 (2012).

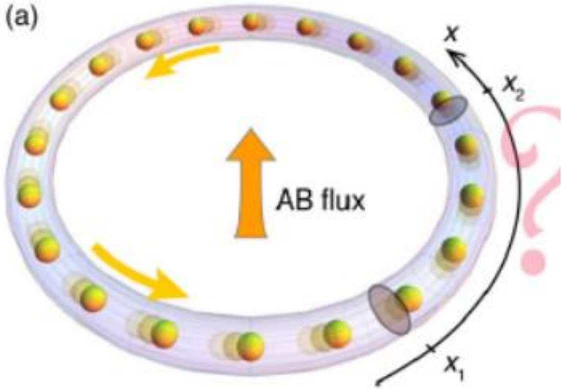
Selected for a Viewpoint in *Physics*
PRL **109,** 160401 (2012) PHYSICAL REVIEW LETTERS week ending 19 OCTOBER 2012

Quantum Time Crystals

Frank Wilczek
Center for Theoretical Physics Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 29 March 2012; published 15 October 2012)

Some subtleties and apparent difficulties associated with the notion of spontaneous breaking of time-translation symmetry in quantum mechanics are identified and resolved. A model exhibiting that phenomenon is displayed. The possibility and significance of breaking of imaginary time-translation symmetry is discussed.

T. Li et al, Space-time crystals of trapped ions,
Phys. Rev. Lett. 109, 163001 (2012).



…..and criticism.

P. Bruno, "*Comment on quantum time crystals*", Phys. Rev. Lett. 110, 118901 (2013).
P. Bruno, *Comment on space-time crystals of trapped ions*, Phys. Rev. Lett. 111, 029301 (2013).
P. Bruno*, Impossibility of spontaneously rotating time crystals: A no-go theorem*, Phys. Rev. Lett. 111, 070402 (2013).
P. Nozieres, *Time crystals: Can diamagnetic currents drive a charge density wave into rotation?* Europhys. Lett. 103, 57008 (2013).
H. Watanabe, and M. Oshikawa, *Absence of quantum time crystals*, Phys. Rev. Lett. 114, 251603 (2015)

Nevertheless the name “Time crystal” has survived: Systems out equilibrium with long-living oscillations are called now “Time crystals”.

Theory….

G. Volovik, *On the broken time translation symmetry in macroscopic systems: Precessing states and off-diagonal long-range order*, JETP Lett. 98, 491 (2013).
V. Khemani, A. Lazarides, R. Moessner, and S.L. Sondhi, *Phase structure of driven quantum systems.* Phys. Rev. Lett. 116, 250401 (2016).
D.V. Else, B. Bauer, and C. Nayak, *Floquet time crystals*, Phys. Rev. Lett. 117, 090402 (2016).
N.Y. Yao, A.C. Potter, I.-D. Potirniche, and A. Vishwanath, *Discrete Time Crystals: Rigidity, criticality, and realizations,* Phys. Rev. Lett. 118, 030401 (2017).

and Experiment.

S. Autti, V.B. Eltsov, and G. E. Volovik, *Observation of a time quasicrystal and its transition to a superfluid time crystal*, Phys. Rev. Lett. 120, 215301 (2018).
J. Zhang, P.W. Hess, A. Kyprianidis, P. Becker, A. Lee, J. Smith, G. Pagano, I.D. Potirniche, A.C. Potter, A. Vishwanath, N.Y. Yao, and C. Monroe, *Observation of a discrete time crystal*, Nature 543, 217-220 (2017).
S. Choi, J. Choi R. Landig, G. Kucsko, H. Zhou, J. Isoya, F. Jelezko, S. Onoda, H. Sumiya, V. Khemani, C. von Keyserlingk, N.Y. Yao, E. Demler E, and M.D. Lukin, *Observation of discrete time-crystalline order in a disordered dipolar many-body system*, Nature 543, 221 (2017).

Nice development but Thermodynamic Quantum Time-Space Crystals are different!

Model of interacting fermions.

Hamiltonian:

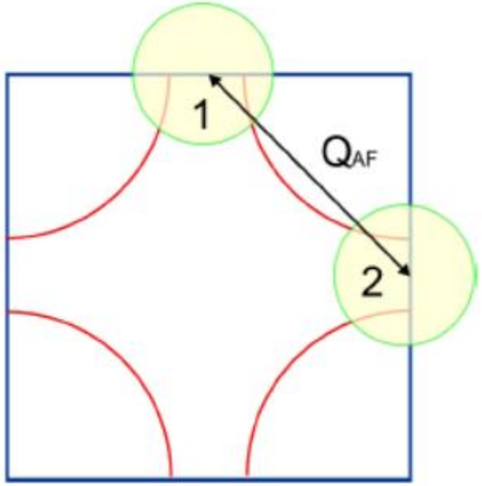
$$\hat{H} = \sum_p c_p^+ \left(\varepsilon_p^+ + \varepsilon_p^- \Sigma_3\right) c_p + \frac{1}{4V}\left[\tilde{U}_0 \left(\sum_p c_p^+ \Sigma_1 c_p\right)^2 - U_0 \left(\sum_p c_p^+ \Sigma_2 c_p\right)^2\right].$$

Fermionic operators: $c_p = \left\{c_p^1, c_p^2\right\},$ V-volume $\varepsilon_p^{\pm} = \frac{1}{2}\left(\varepsilon_1(\mathbf{p}) \pm \varepsilon_2(\mathbf{p})\right)$

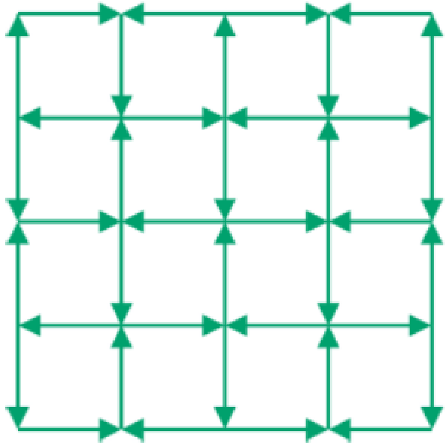
Spectrum: $\varepsilon_a(\mathbf{p})$ Bands: a=1,2 $\Sigma_1, \Sigma_2, \Sigma_3$ -Pauli Matrices

$\tilde{U}_0 \geq U_0$ >0 -crucial for existence of the new state!

Two bands:



(a) Fermi surface and interaction.


(b) Loop currents.

Loop currents at $\tilde{U}_0 = 0$

P.A. Volkov, K.B. Efetov (2018)

Thermodynamics and calculations in imaginary time τ

Method of functional integration in imaginary time.

$$Z = \int \exp\left[-S_0[\chi] - S_{\text{int}}[\chi]\right] D\chi,$$

$$S_0[\chi] = \int_0^{1/T} \chi_p^+(\tau)\left(\partial_\tau + \varepsilon^+(\mathbf{p}) - \varepsilon^-(\mathbf{p})\Sigma_2\right)\chi_p(\tau)\, d\tau,$$

$$S_{\text{int}}[\chi] = -\frac{1}{4V}\int_0^{1/T}\left[U_0\Big(\sum_p \chi_p^+(\tau)\Sigma_3\chi_p(\tau)\Big)^2 - \tilde{U}_0\Big(\sum_p \chi_p^+(\tau)\Sigma_1\chi_p(\tau)\Big)^2\right] d\tau$$

$$\chi_p(\tau) = \left(\chi_p^1(\tau), \chi_p^2(\tau)\right)$$

$$\chi(\tau) = -\chi(\tau + 1/T).$$ Fermionic boundary conditions.

Hubbard-Stratonovich Transformation.

$$F = -T\ln\left[\int \exp\left[-\frac{\mathcal{F}[b,b_1]}{T}\right] Db\,Db_1\right].$$

$$b(\tau) = b(\tau + 1/T), \qquad b_1(\tau) = b_1(\tau + 1/T)$$ Bosonic boundary conditions

$$\frac{\mathcal{F}[b,b_1]}{T} = \int_0^{1/T}\left[-2\sum_{\mathbf{p}} \mathrm{tr}\left[\ln\left(h(\tau,\mathbf{p}) - ib_1(\tau)\Sigma_1\right)\right]_{\tau,\tau} + V\left(\frac{b^2(\tau)}{U_0} + \frac{b_1^2(\tau)}{\tilde{U}_0}\right)\right] d\tau,$$

$$h(\tau,\mathbf{p}) = \partial_\tau + \varepsilon^+(\mathbf{p}) - \varepsilon^-(\mathbf{p})\Sigma_2 - b(\tau)\Sigma_3$$

In the limit $V \to \infty$ it is enough to minimize the free energy functional $\mathcal{F}[b,b_1]$!

Equation at $b_1(\tau)=0$

$$b(\tau) = -U_0 \mathrm{tr}\int \Sigma_3 \left[h^{-1}(\tau,\mathbf{p})\right]_{\tau,\tau} \frac{d\mathbf{p}}{(2\pi)^2},$$

$$b(\tau)=\gamma, b_1(\tau)=0$$

Exact Solutions.

$$b_0(\tau) = k\gamma \mathrm{sn}\left(\gamma(\tau-\tau_0)\,|\,k\right)$$

DDW-like solution

(P.A.Volkov, K.B. Efetov, 2018)

sn(u)-Jacobi elliptic function

Equation for γ

S.I. Mukhin, (2009, 2011)

(For polymers in coordinate space: S.A. Brazovskii, S.A. Gordyunin, N.N. Kirova (1980)).

$$1 = U_0 \int_0^{1/T} \frac{1}{E(\mathbf{p})} \frac{d\mathbf{p}}{(2\pi)^2},$$

$$E(\mathbf{p}) = \sqrt{\left(\varepsilon^-(\mathbf{p})\right)^2 + \gamma^2}$$

$$\gamma = 4K(k)mT$$

Valid for $k \to 1$

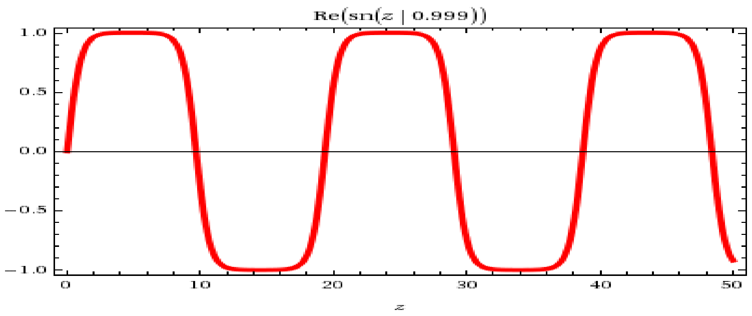
BCS-like equation

Elliptic function $sn(x,k)$ (short information).

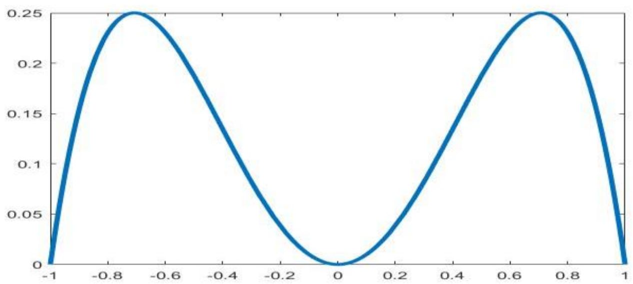
At k=0 $sn(x,0)=\sin x$

At k=1 $sn(x,1)=\tanh x.$

$sn(x,k)\to\tanh x$ as $k\to 1$



Function sn(x,k) at $k^2=0.999$



Classical motion in the potential x^2-x^4

The most important properties of the elliptic functions:
double periodicity in the complex plane.

The function sn(x,k) has the period 4K(k) along the real axis x and 2K(k') along the imaginary one.
K(k)- first kind elliptic integral, and $k^2 + k'^2 = 1$.

The solution of the equation containing only the order parameter b in terms of the function sn(x,k) is a chain of instantons-anti-instanton pairs.

The periodicity condition $b(\tau) = b(\tau + 1/T)$ implies $\gamma = 4mK(k)T$, m-integer.

The solution of the mean field equations at $b_1(\tau) = 0$ in terms of the elliptic function always exists as soon as the periodicity condition is fulfilled!

However, it turns out that the homogeneous solution is always energetically favorable for the case $b_1(\tau) = 0$!

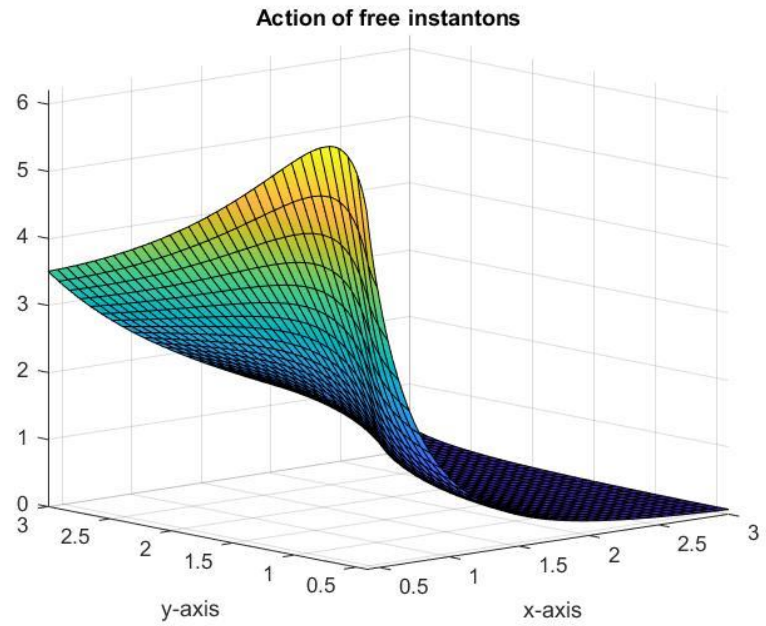
Results of calculations of free energy.

Part I . The charge variable $b_1(\tau)$ is neglected.
Only the loop current order parameter $b(\tau)$ is taken into account.

Numerics is done choosing:

$$\varepsilon^{-}(\mathbf{p})=\frac{\alpha+\beta}{2}\left(p_x^2-p_y^2\right)+P(T),$$

$$x=P(T)/\gamma$$
$$y=\Lambda/\gamma$$



ΔS_0 is always positive!

In order to find the correct solution corresponding to the minimum of the free action one should solve both the equations for $b(\tau)$ and $b_1(\tau)$ but exact solution is hardly possible.

Part II. Instead, the functional of the free energy is expanded in the field $b_1(\tau)$ up to linear and quadratic terms. These fields are integrated out leading to a new term in the action for the field $b(\tau)$ containing time derivatives of $b(\tau)$.
(The field $b_1(\tau)$ plays a role of a “gluon”).

$$\Delta F = F_{\text{inst}} + F_{\text{II}}.$$

$$\frac{\mathcal{F}_1[b_0, b_1]}{VT} = -2J\int_0^{1/T} \dot{b}_0(\tau) b_1(\tau)\, d\tau,$$

$$\frac{F_{\text{inst}}}{2mVT} = \int\left[\ln\frac{E(\mathbf{p})+\gamma}{E(\mathbf{p})-\gamma} - \frac{2\gamma}{E(\mathbf{p})}\right]\frac{d\mathbf{p}}{(2\pi)^2},$$

Energy of instantons

$$\frac{\mathcal{F}[b, b_1]}{T} = \int_0^{1/T}\left[-2\sum_{\mathbf{p}}\mathrm{tr}\left[\ln\left(h(\tau,\mathbf{p}) - ib_1(\tau)\Sigma_1\right)\right]_{\tau,\tau} + V\left(\frac{b^2(\tau)}{U_0} + \frac{b_1^2(\tau)}{\tilde{U}_0}\right)\right]d\tau,$$

“Bare energy” of instanton.

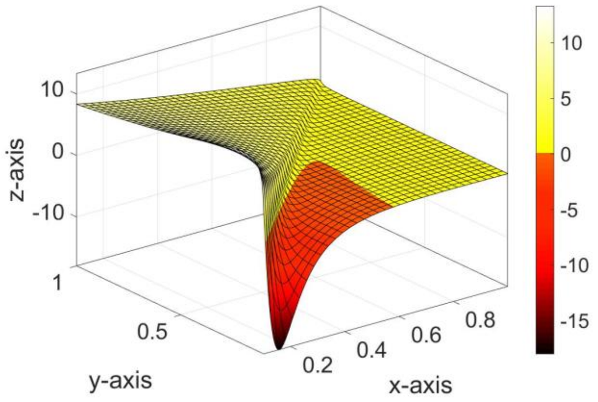
$$\frac{F_{\text{II}}}{2mVT} = -\frac{\tilde{U}_0}{4}\left[\int\frac{sgn(\varepsilon(\mathbf{p}))}{E(\mathbf{p})\sqrt{\left((\varepsilon^-(\mathbf{p}))^2 + \frac{\gamma^2(1-k)^2}{4}\right)}}\frac{d\mathbf{p}}{(2\pi)^2}\right]^2$$

Effective attraction.

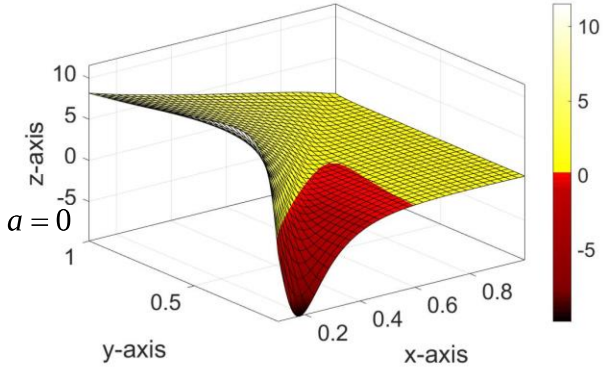
Screening (second order in $b_1(\tau)$) should also be accounted for.

$$a = \frac{U_0}{\tilde{U}_0}$$

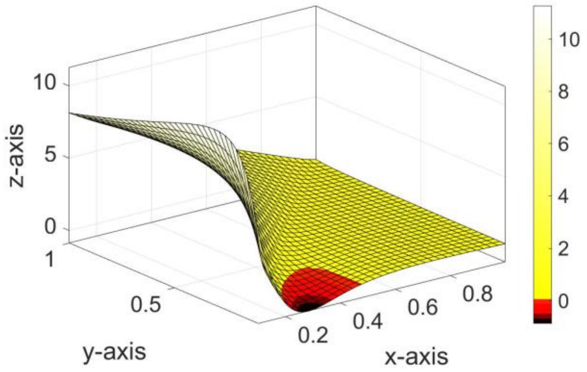
Numerical study of the total action of instantons-antiinstantons.



$k = 0.99$

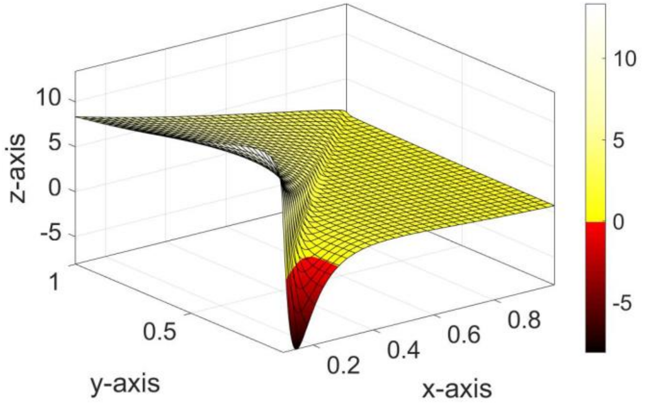


$k = 0.90$

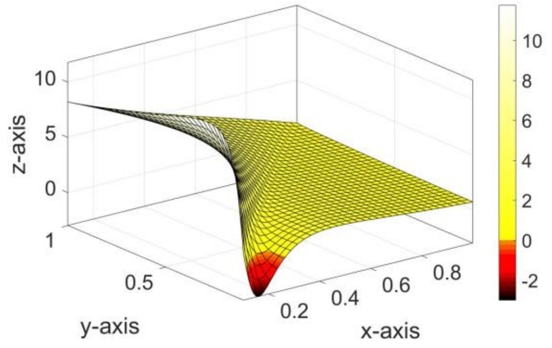


$k = 0.70$

$a = 0$



$k = 0.99$



$k = 0.90$

$a = 1$

Thermodynamic Time-Space Crystal.

$$C\left(t_1;t_2\right)=\frac{1}{V^2}\langle\tilde{\Phi}_0^*\,|\,\hat{A}\left(t_1\right)\hat{A}\left(t_2\right)\left|\tilde{\Phi}_0\right\rangle,$$

$$\hat{A}\left(t\right)=e^{i\hat{H}t}\hat{A}e^{-i\hat{H}t}$$

$$\hat{A}=U_0\sum_p \mathrm{tr}\left(c_p^+\Sigma_3 c_p\right)$$

$$c_p\left(t\right)=e^{i\hat{H}t}c_p e^{-i\hat{H}t}$$

If $\tilde{\Phi}_0$ is time-independent, no-time crystal (in agreement with "no-go" theorem).

<u>However, the time translation symmetry breaks down at a critical point and order parameter $B\left(t\right)=-ib\left(it\right)$ appears!</u>

Then, $\tilde{\Phi}_0\to\Phi_{mf}\left(t-t_0\right)$, t_0 -is arbitrary

and $$C\left(t_1-t_2\right)=\int\left\langle\Phi\left(t_1-t_0\right)\right|A\left|\Phi\left(t_1-t_0\right)\right\rangle\left\langle\Phi\left(t_2-t_0\right)\right|A\left|\Phi\left(t_2-t_0\right)\right\rangle dt_0$$

$$\int\left\langle\Phi\left(t-t_0\right)\right|A\left|\Phi\left(t-t_0\right)\right\rangle dt_0=0$$

Thermodynamic Time-Space Crystal.

Elliptic functions are periodic along both real and imaginary axes:

Periodic behavior of correlation functions also along real time!

$$N\left(t_1-t_2\right)=-\overline{b\left(i\left(t_1-t_0\right)\right)b\left(i\left(t_2-t_0\right)\right)}$$

Bar stands for averaging over t_0

$$N\left(t\right)\approx 2\gamma^2\sum_{n=1}^{\infty} f_n^2\cos\left(2\gamma nt\right),\quad f_n=\left[1-\frac{1}{2}\left(\frac{1-k}{8}\right)^{2n}\right]$$

$$1-k\ll 1$$

$$\overline{b\left(t-t_0\right)}=0 \Rightarrow$$

No time-dependent currents and no radiation!

Nondecaying oscillations!!!
Perfect qubit?

Fourier transform $N(\omega)$ of the function $N(t_1 - t_2)$ (Limit of small 1-k)

$$\chi(\omega,\mathbf{q}) = \chi_0 \sum_{n=1}^{\infty} f_n \delta(\omega - 2n\gamma)\delta(\mathbf{q} - \mathbf{Q}_{AF}),$$

2γ is the gap!

No peak at zero frequency $\Longrightarrow$ no static order at (π,π) !

Determines peaks in neutron inelastic scattering at Q=(π,π).

The function $N(\omega)$ has equidistant δ -peaks $\Longrightarrow$ corresponds to the space- time crystals!

Conclusions.

Thermodynamic quantum space-time crystals may exist as soon as one obtains a time-dependent order parameter (the “no-go ” theorem is not applicable to this case).

One can expect a completely new type of a phase transition.

As the time average of the order parameter is zero, one does not expect *perpetuum mobile* (in agreement with energy conservation).

A new proposal for the Pseudogap state in cuprates (time reversal symmetry is broken but static magnetic moments at (π,π) do not exist, possibility of peaks at 2γ in inelastic neutron scattering).