Tribute to Lev Gor'kov

Eternal Quest for Knowledge, Gor'kov (1961)

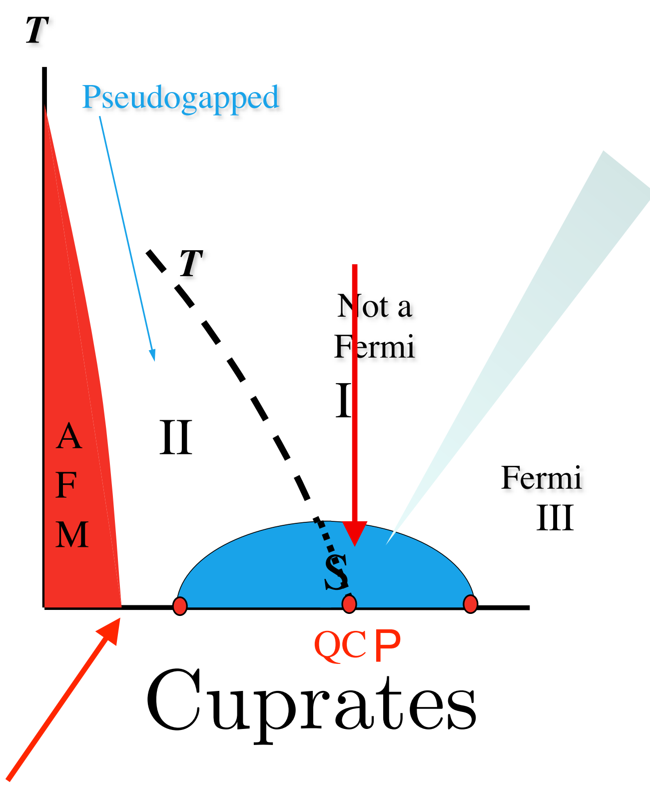
Freedom of Space and time metrics.

Quantum critical fluctuations of the dissipative Qtm. XY model.

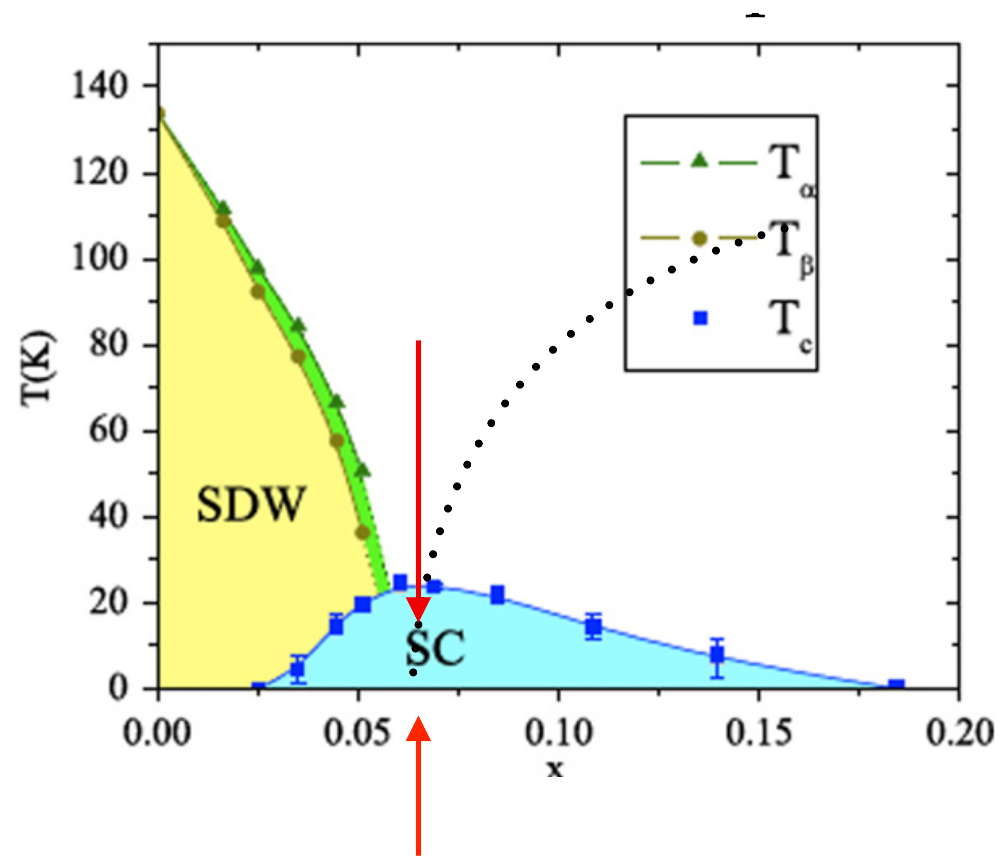
Possible Applications to FM, AFM and Cuprate Qtm. Criticality,
and Superconductor-Insulator-metal Qtm. transitions

I will speak mostly about properties down the red-line

Same! Same?



AFM -QCP



AFM -QCP

Also, same properties in quasi-2D Heavy-Fermions,
YbRh(2)Si(2) - Paschen, Steglich, et al.
CeCu(6) - Lohneysen et al.
CeCoIn(5)- Thompson et al.

The observed qtm. critical properties cannot be understood by any model which is in the class of extension of Wilson-Fisher type models to qtm. dynamics.

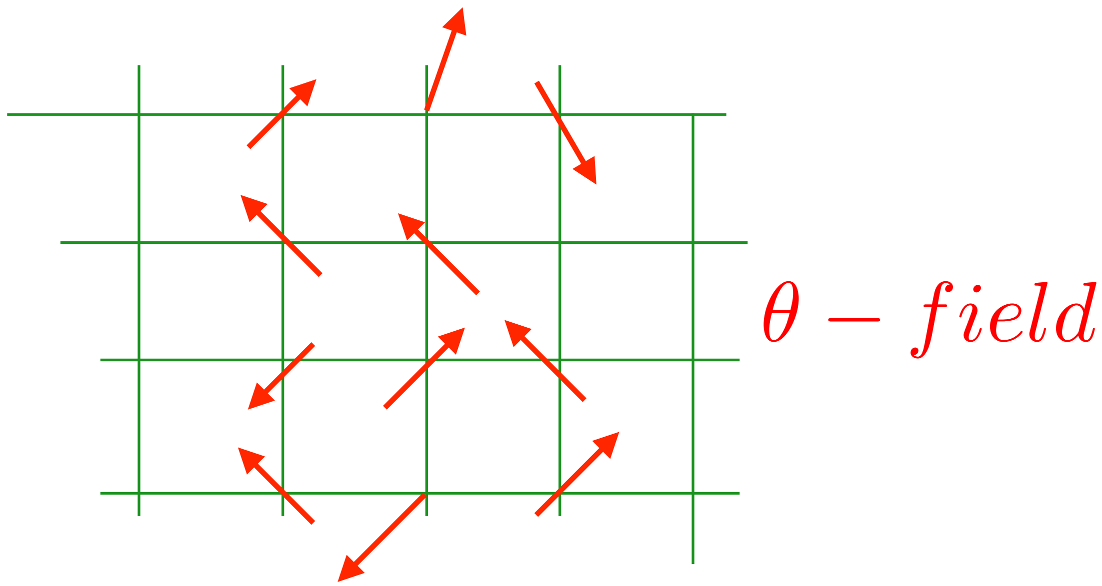
Hertz, Moriya, ...

- Dynamical critical exponents etc.: d(eff) = z + d.

Quantum-Critical Fluctuations of the Model

(Vivek Aji, CMV - PRL 2007, PRB-2009, 2010)

Classical Model: XY model with 4-fold Anisotropy

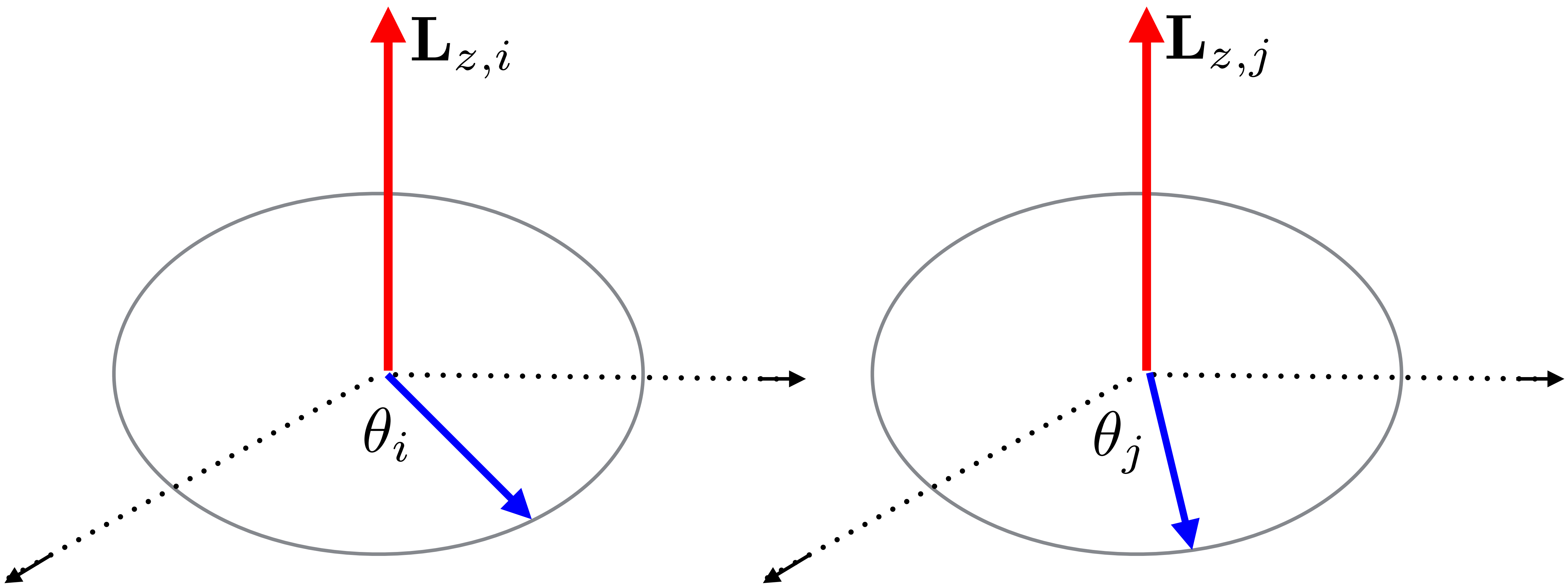


$$\mathcal{L} = \sum_{<ij>} K \cos(\theta_i - \theta_j) + K_4 \cos 2(\theta_i - \theta_j) + h_4 \cos(4\theta_i)$$

Anisotropy: Marginally Irrelevant in the Fluctuation region, Highly relevant in the ordered region. (Ashkin-Teller Model)

Topological Phase Transition (Kosterlitz-Thouless, Berezinsky Ordering by Binding of vortices of opposite circulation.

Quantum XY - Model coupled to Fermions.



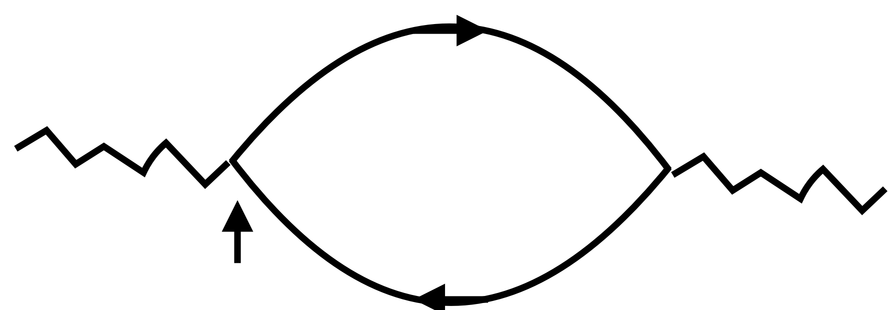
$$H = \sum_i \frac{\mathbf{L}_{z,i}^2}{2I} + J \sum_{i,j} \cos(\theta_i - \theta_j) + \mathrm{Diss.}$$

Phase transition driven by topological defects:
warps and vortices, not by anharmonic oscillations.

The Qtm. model is almost as well soluble as the classical model.

Dissipation :

$$\frac{\alpha}{4\pi}\omega q^2 |\theta(q,\omega)|^2$$



$j \propto \nabla\theta$

Usual way of thinking of the problem:
vortex loops in space and imaginary time.

Not soluble in a controlled way.

New variables needed?

Solution of the Model

1. Analytical solution: (Aji-CMV -prl2007, prb2009, Hou- prb2016).

   Find an exact transformation from θ to orthogonal topological excitations

   $\rho_v(\mathbf{r},\tau)$ and $\rho_w((\mathbf{r},\tau)$

   vortices and warps.

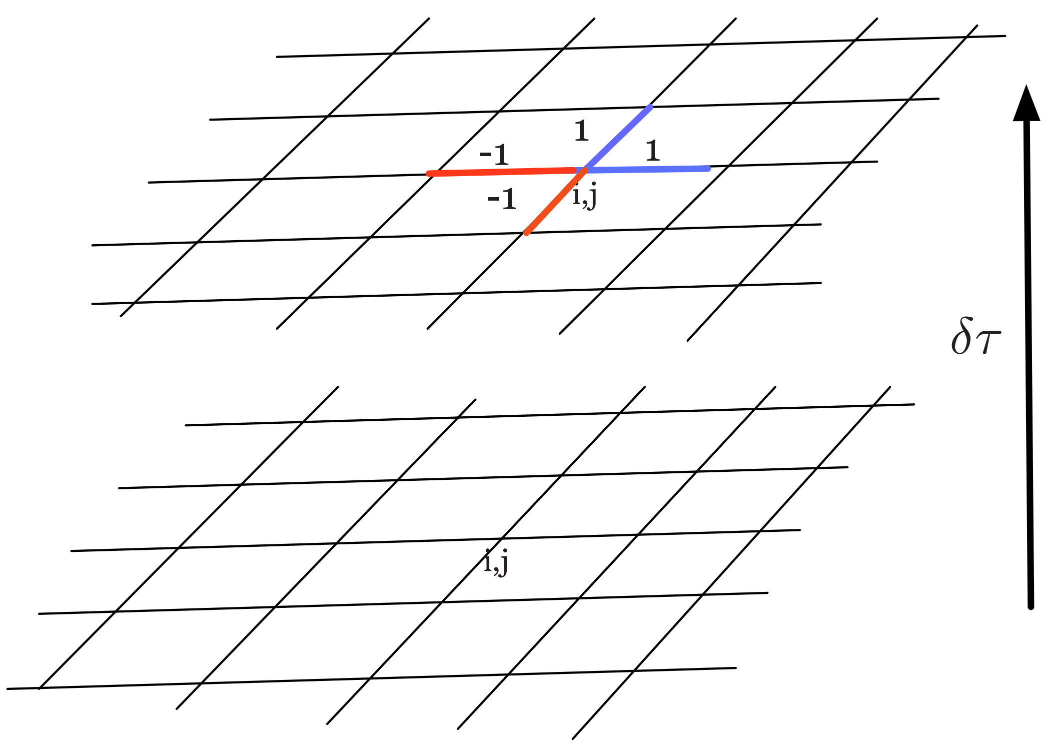
2. Quantum-Monte-carlo calcs.

$$\mathbf{m}_{ij,\tau,\tau'} = (\theta)_{i,\tau} - (\theta)_{j,\tau'}, \quad \rho_v = \nabla \times \mathbf{m}.$$

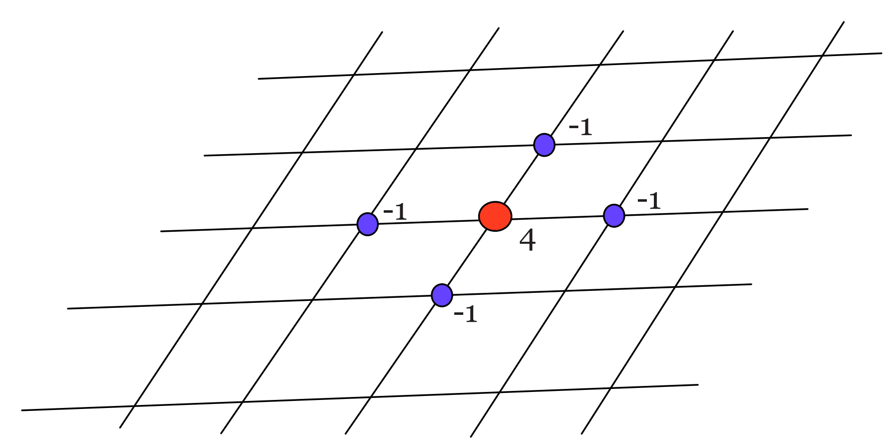
What is a warp?

Jump in Phase by 2π at a point in space between two time-slices,

Change in $\mathbf{m}$:



Change in $\nabla \cdot \mathbf{m}$:



Creates a monopole of charge 4 surrounded by 4 monopoles of charge -1.

In terms of these variables, a miracle:

(Aji,CMV (2009)

$$S = K \int d\tau d\mathbf{r} d\mathbf{r}' \ln|\mathbf{r}-\mathbf{r}'| \rho_v(\mathbf{r},\tau)\rho_v(\mathbf{r}',\tau)$$

$$+ \frac{\alpha}{4\pi} \int d\tau d\tau' d\mathbf{r} \ln(\tau-\tau')\rho_w(\mathbf{r},\tau)\rho_w(\mathbf{r},\tau')$$

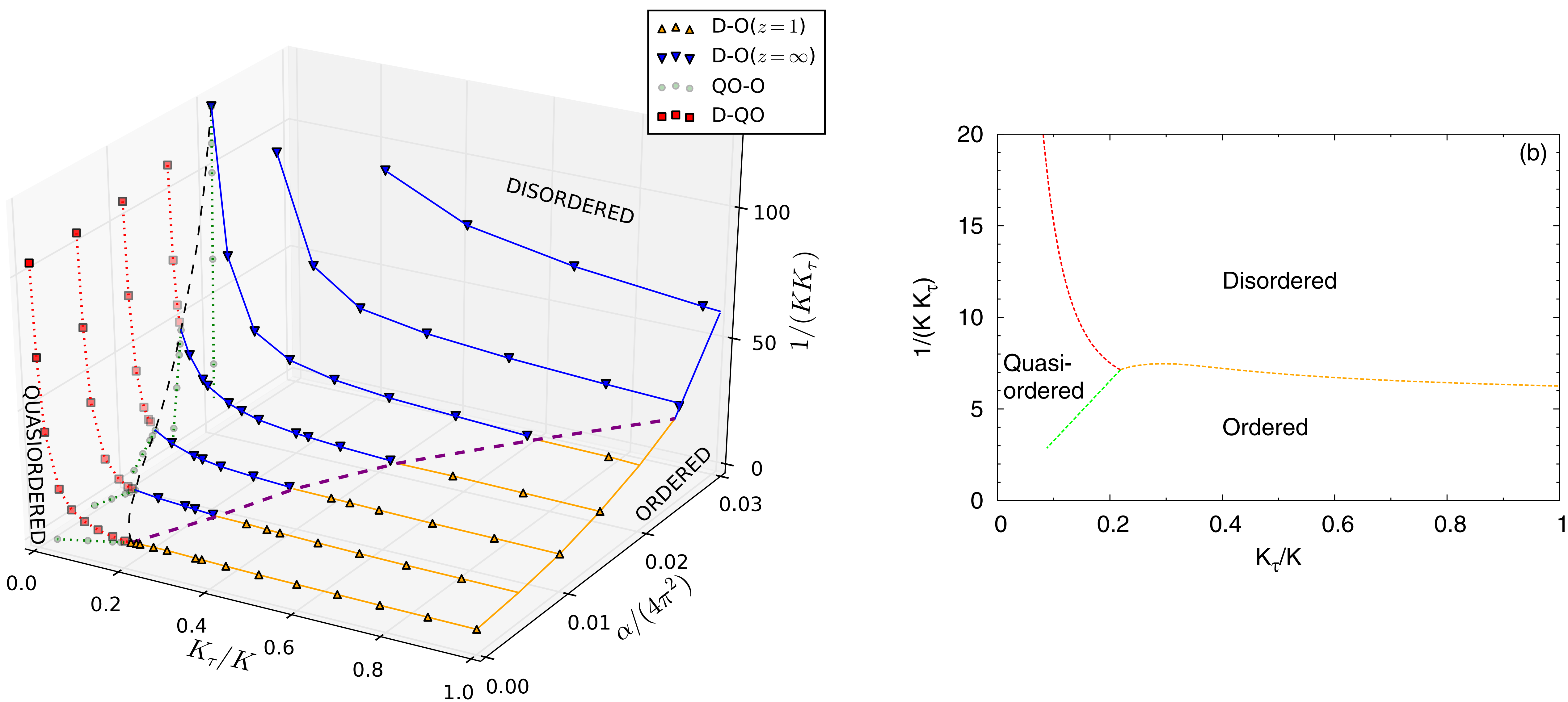
$$+ \int d\tau d\tau' d\mathbf{r} d\mathbf{r}' \frac{K'}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + v^2(\tau-\tau')^2}} \rho_w(\mathbf{r},\tau)\rho_w(\mathbf{r}',\tau').$$

$$K' = \sqrt{K K_\tau}, \quad v^2 = \frac{K}{K_\tau}.$$

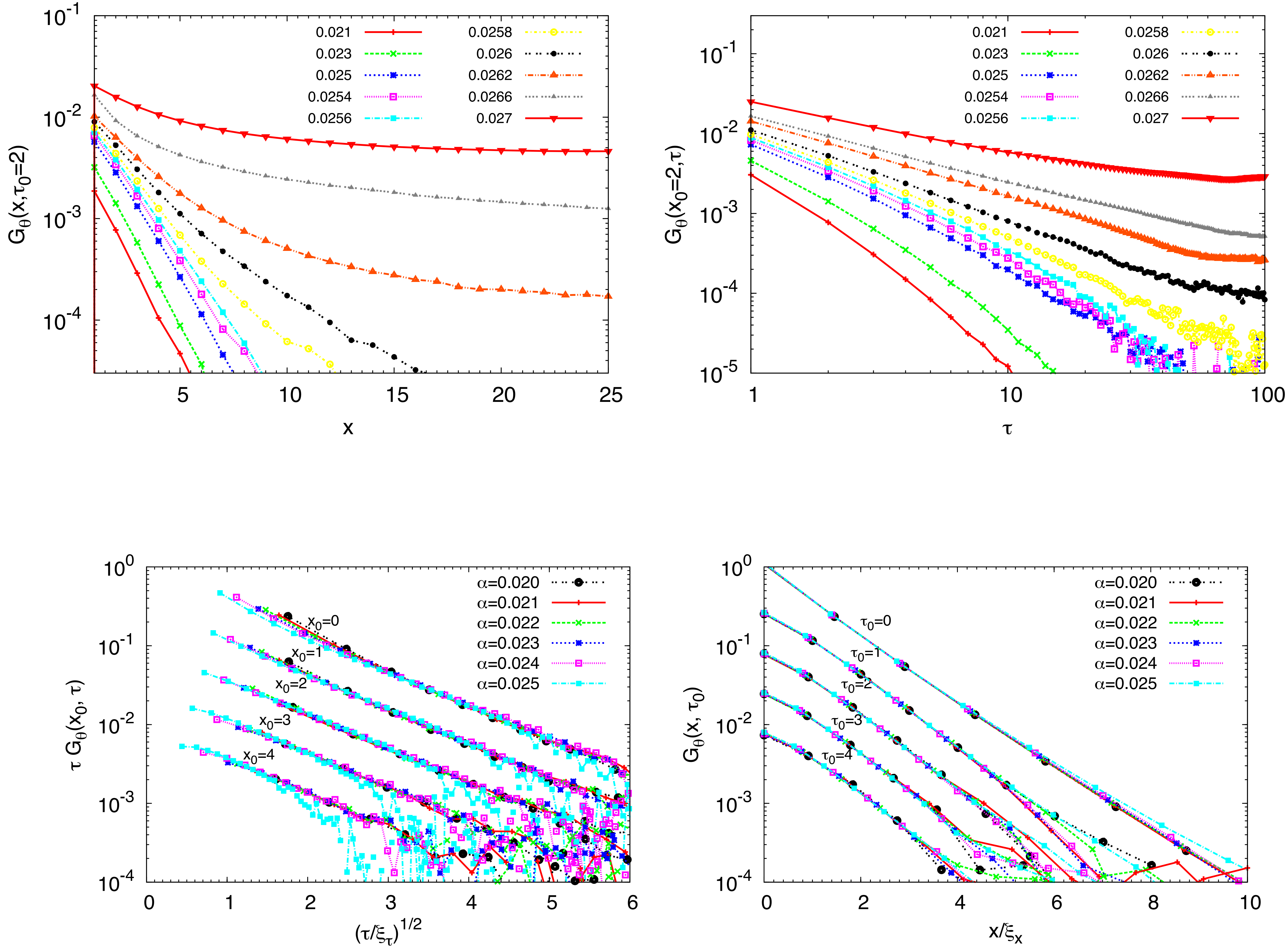
RG on This form of S:
ρ_v and ρ_w are orthogonal.
The third term is less singular than the first two,
which are equivalent to $v \to 0$, or $v \to \infty$.
But v is relevant both around $v \to 0$ and $v \to \infty$.

Calculated Phase Diagram and Correlation functions
tested by Quantum Monte-Carlo calculations.

Phase Diagram



$$G(x,\tau) = \; < e^{i\theta(x,\tau)} \; e^{-i\theta(0,0)} >$$



Order parameter correlations:

$$G_{(\cos\theta)}(x,\tau) = G_0 \frac{1}{\tau} e^{-\tau/\xi_\tau} e^{-x/\xi_x} \log(a/x)$$

$$\mathrm{Im}\, G_{(\cos\theta)}(q,\omega) = G_0 \tanh\left(\frac{\omega}{\sqrt{(2T)^2+\xi_\tau^{-2}}}\right) \frac{1}{q^2+\xi_x^{-2}}$$

Three remarkable features:

a. Separable function of space and time!

b. "Temperature"-Fourier Transform of $1/\tau$:tanh$(\omega/2T)$

i.e. Quantum-critical Flucts. proposed (1989) for MFL.

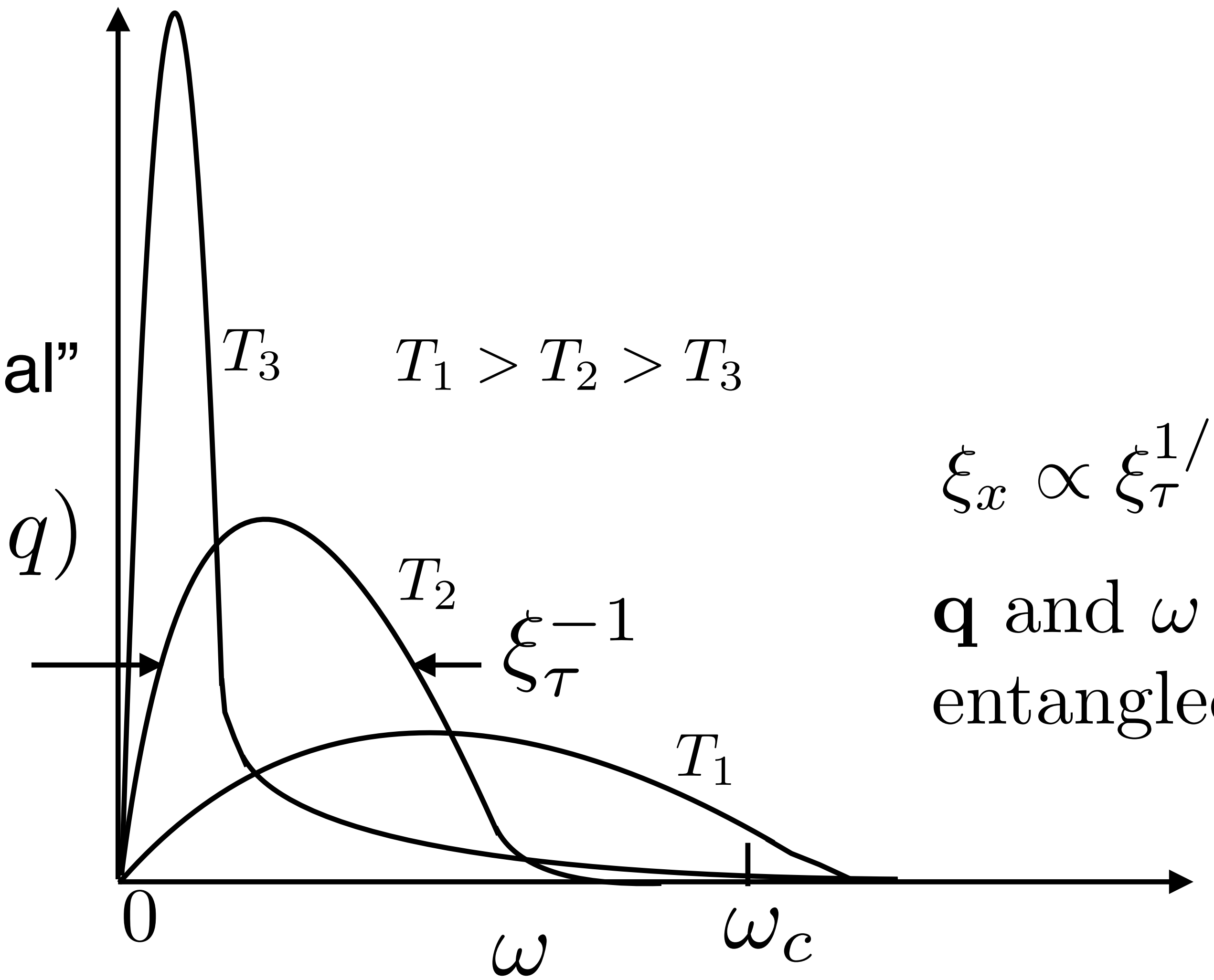
c. Spatial length Scale is log of Temporal length scale

$$\xi_x \propto \log \xi_\tau$$

Contrast with “conventional” Qtm. Crit. Spectra

“Conventional”

- $\mathrm{Im}\ G(\omega, q)$

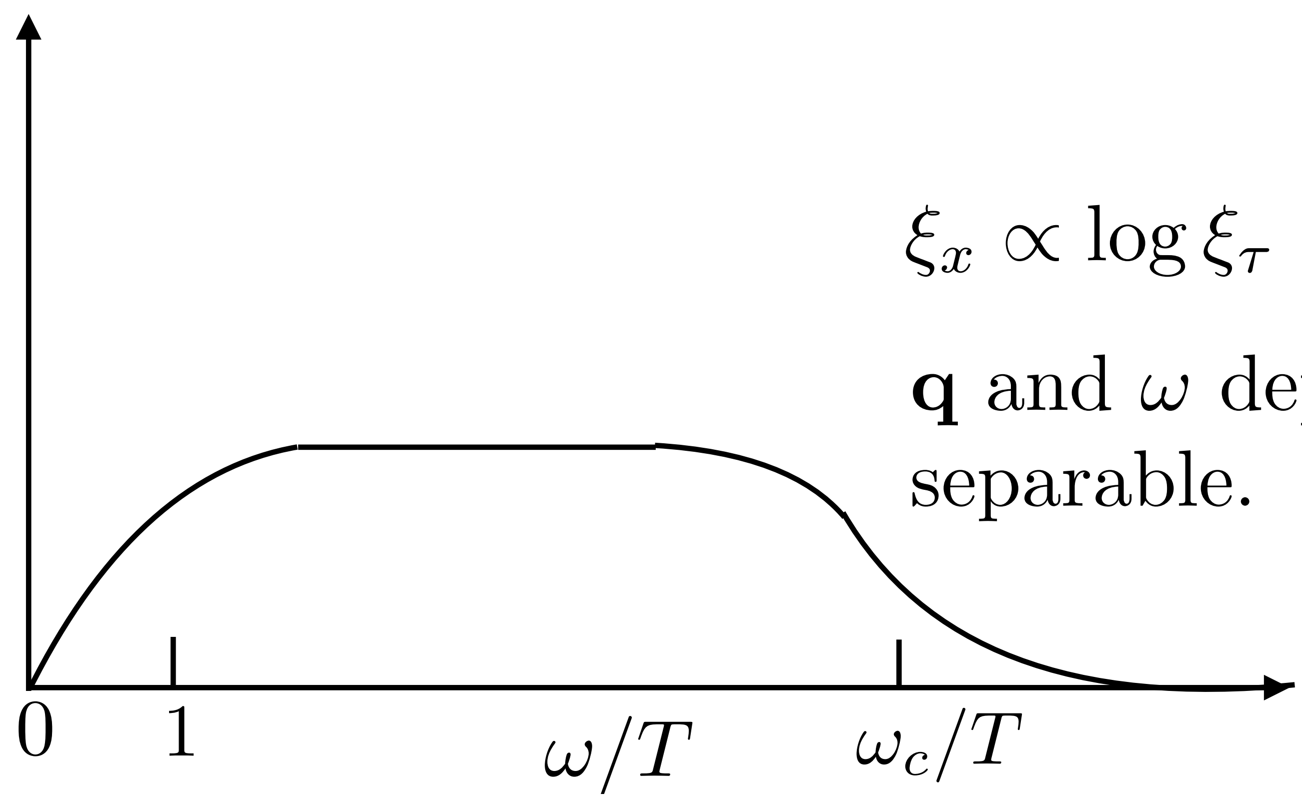


$\xi_x \propto \xi_\tau^{1/z}$

q and ω dependence entangled.

Diss XY

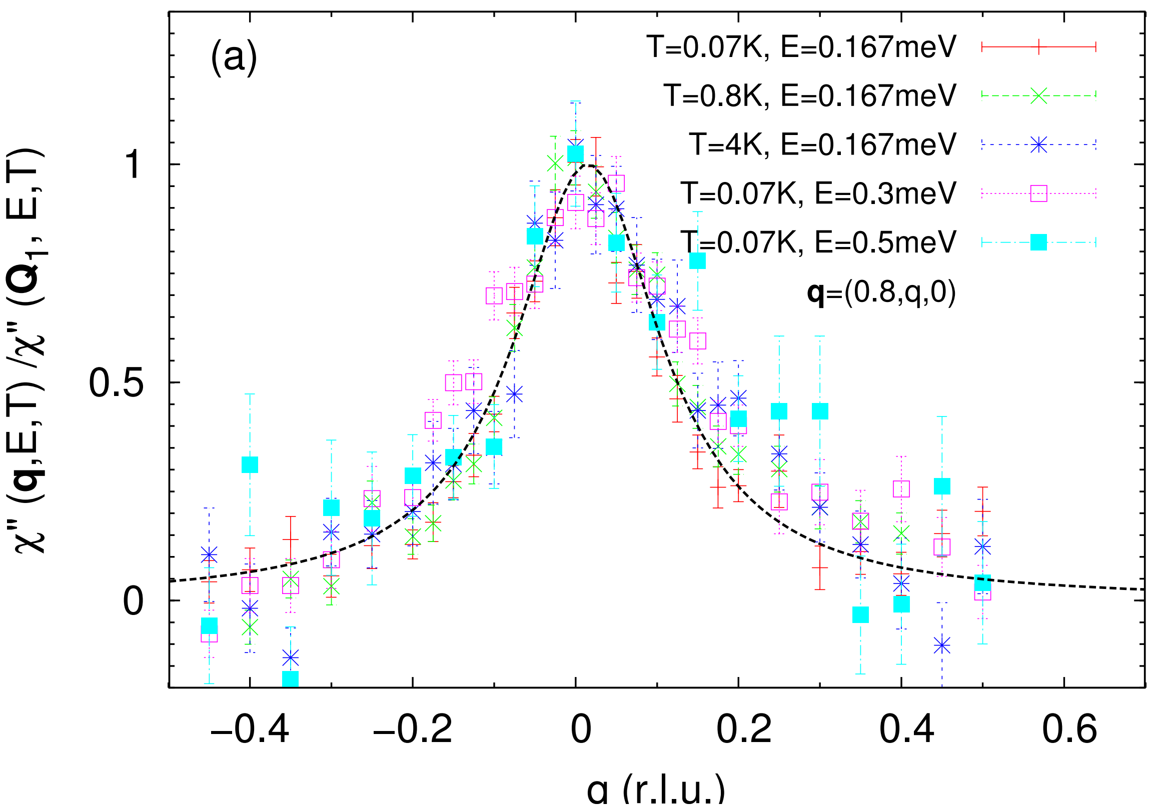
$-Im\ G(\omega, q)$

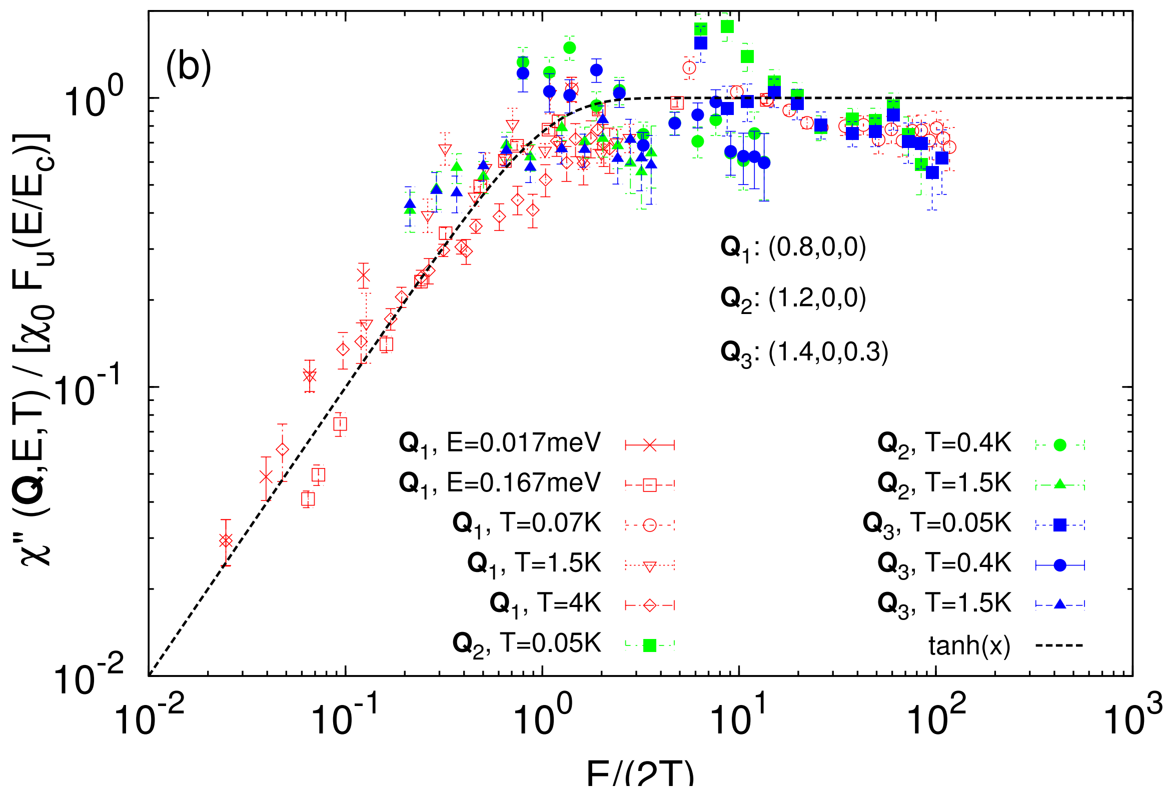


$\xi_x \propto \log \xi_\tau$

q and ω dependence separable.

$CeCu_{5.8}Au_{0.2}$
A. Schr$\ddot{o}$der et al., 2001.
(Re-plotted).



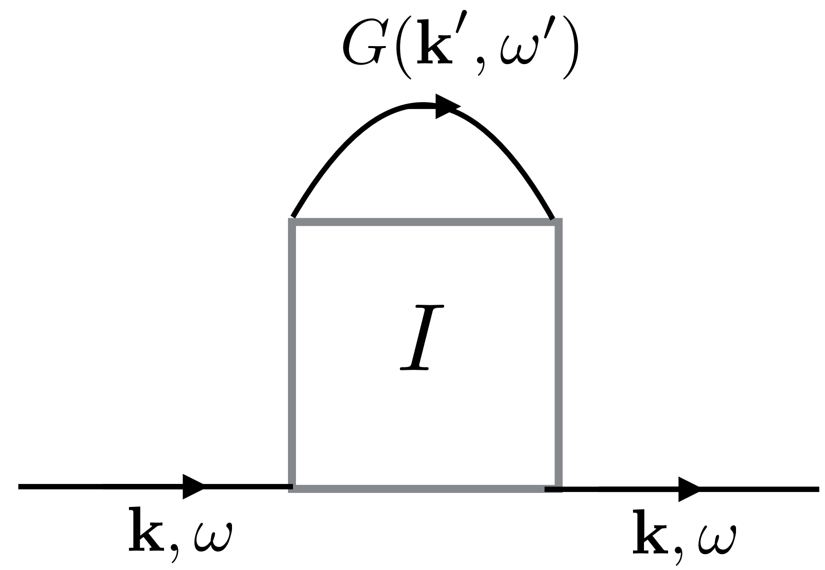


Calculation of measurable properties:
Single particle self-energy, specific heat,
density correlations, resistivity.

The most convenient way:
Regard the fluctuations as an irreducible vertex:

$$I = \frac{\delta\Sigma}{\delta G} \rightarrow$$

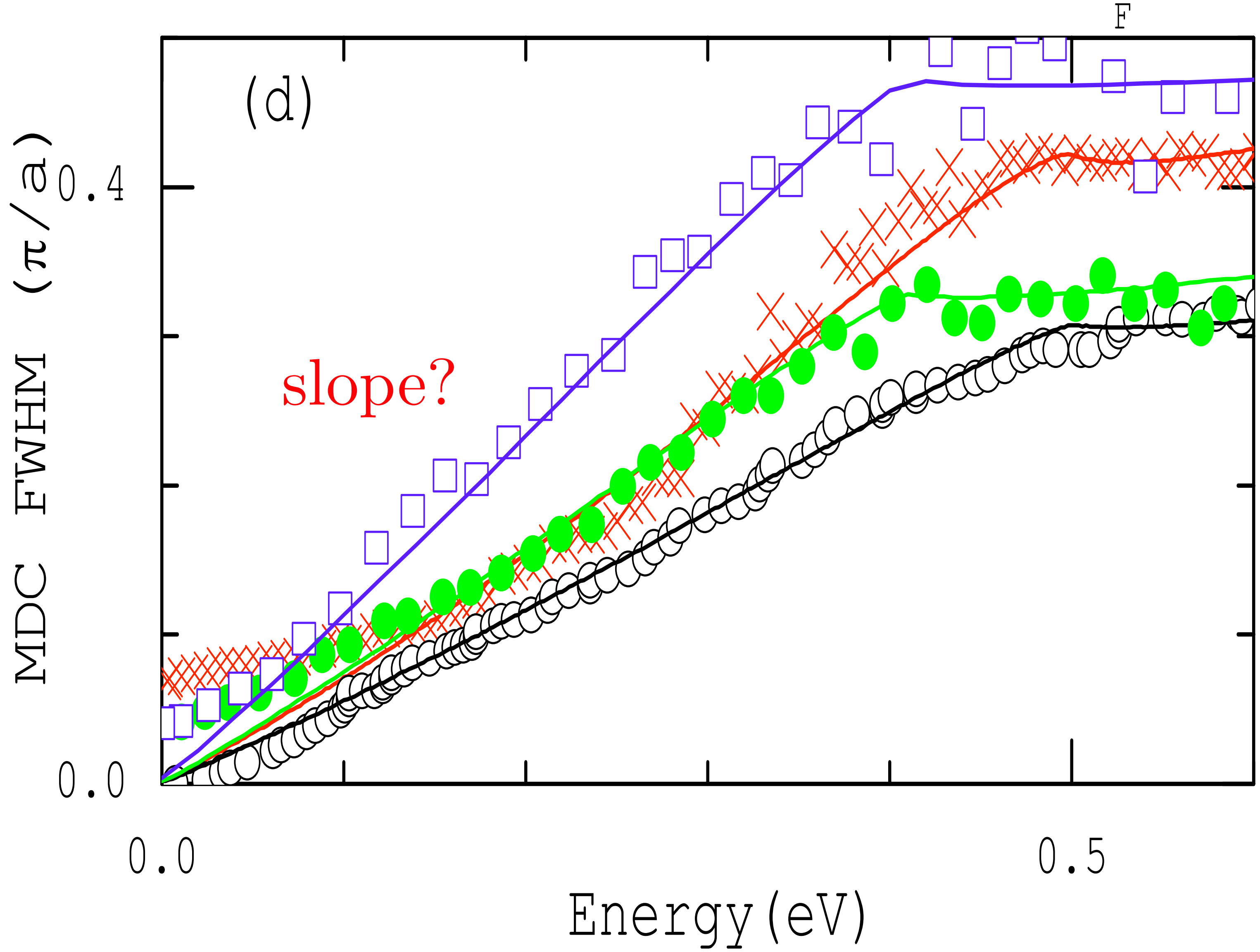
$$\Sigma(\mathbf{k}, \omega) \;\; =$$



ARPES results (2000-2016) for scattering rate at the Fermi-surface

Bi2212 - nodal direction (Lanzara et al.)
Bi2201 - nodal direction (Shen et al.)
LSCO - two directions (Chang et al.)

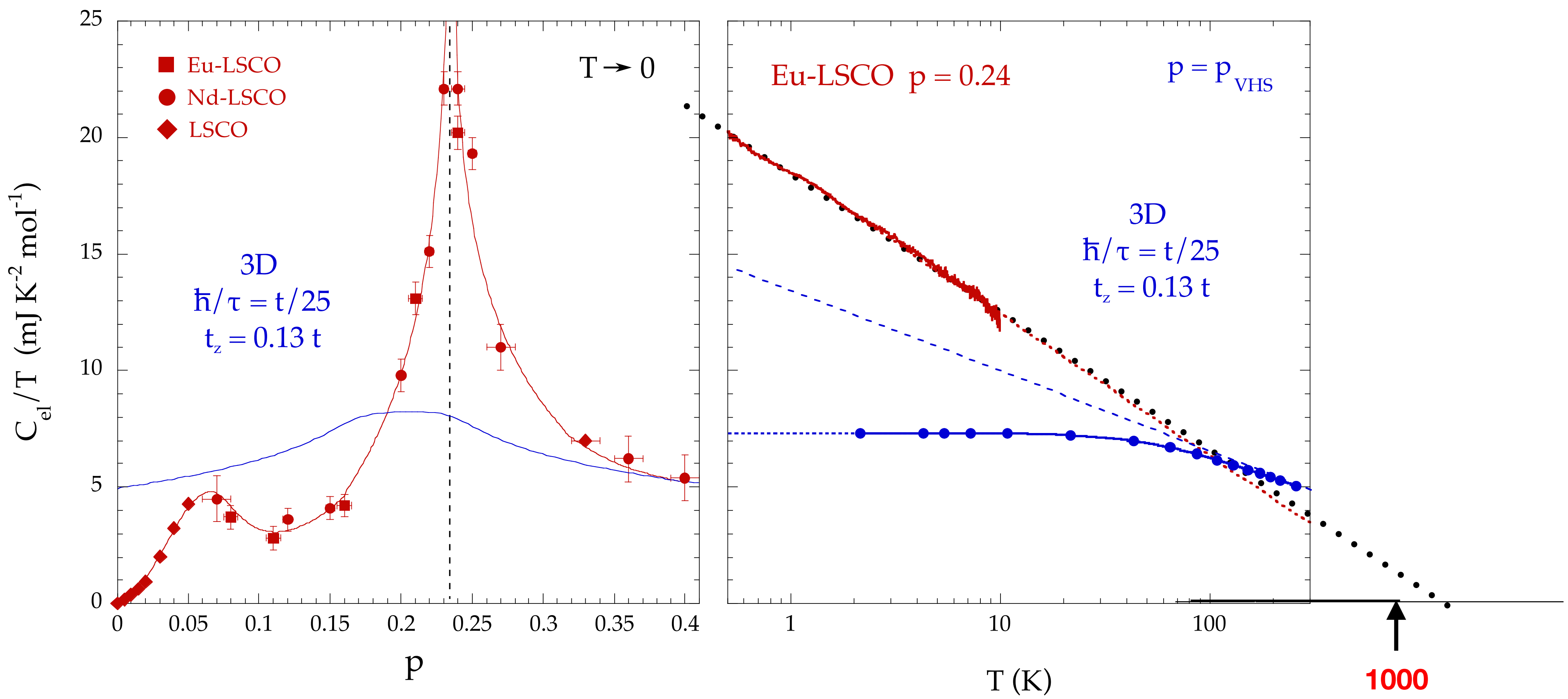
$$\frac{Im\ \Sigma(p_F,\theta,\omega)}{v_F(\theta)} =$$



$$\Rightarrow \quad T_x = 3-5\times 10^3 K$$

g between 0.4 and 0.5

Specific Heat: Michon et al. (2018).



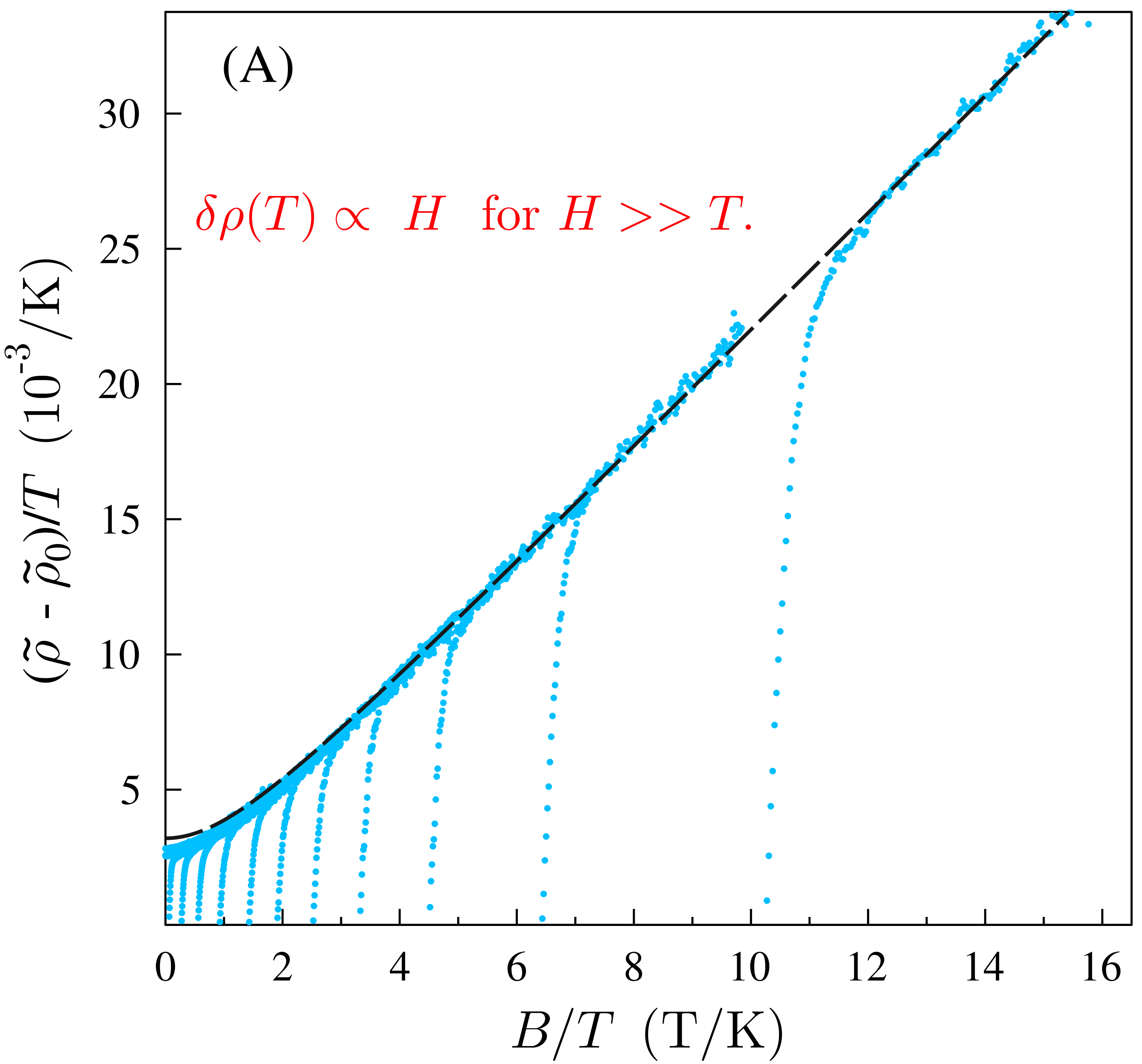
$$C_v/T = A\ g\big(1 + \ln(\tfrac{T_x}{T})\big)$$

$$T_x \approx 2 \times 10^3 K, \quad g \approx 0.5$$

Predicted relation between g here and in scattering rate obeyed as does Tx.

$B_z - T$ scaling in $BaFe_2(As_{1-x}P_x)_2$ at $x = 0.31$

Hayes et al., Nat. Phys. (2016)



Similar results also in Cuprates in quantum-crit. region.

Magnetic Field Dependence of Quantum-crit. Properties.

$$S_B = \sum_i \int_0^\beta d\tau \, \mathbf{B} \cdot \mathbf{L}_{iz}(\tau),$$

Scaling dimensions:
$[B_z][L_z]/[T]$ dimensionless.

Have shown that $[L_z] = 0$
Therefore $[B_z]/[T]$ dimensionless.

It follows that critical properties are homogeneous functions of B/T, with log. corrections.

Tested by Montecarlo calculations.

Microscopic Theory

Coupling of Magnetic fields to XY-model?

Topological Excitations: vortices and warps.

Obvious coupling to vortices:
To orbital angular momentum in charged systems.

But dominant fluctuations are warps, not vortices.

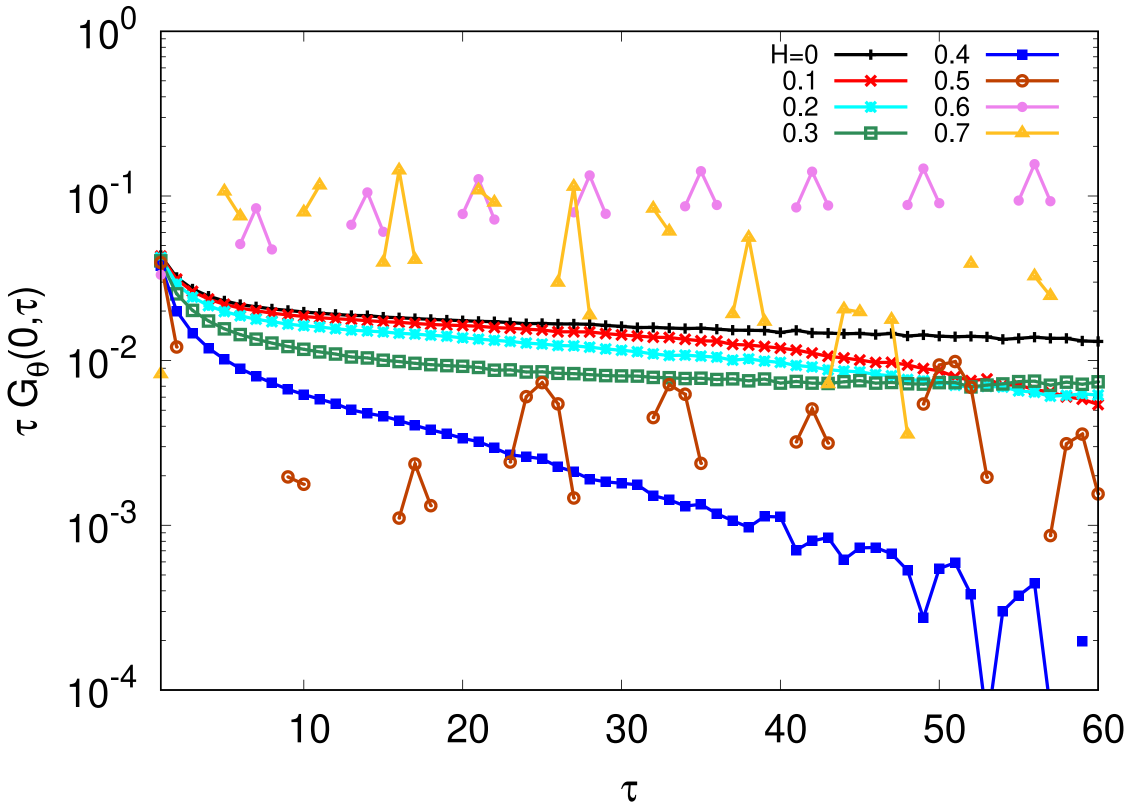
But is there coupling to
intrinsic angular momentum:
$\hat{\mathbf{z}} \cdot \mathbf{B} \ \int_0^\beta i \frac{\partial \theta}{\partial t}$?

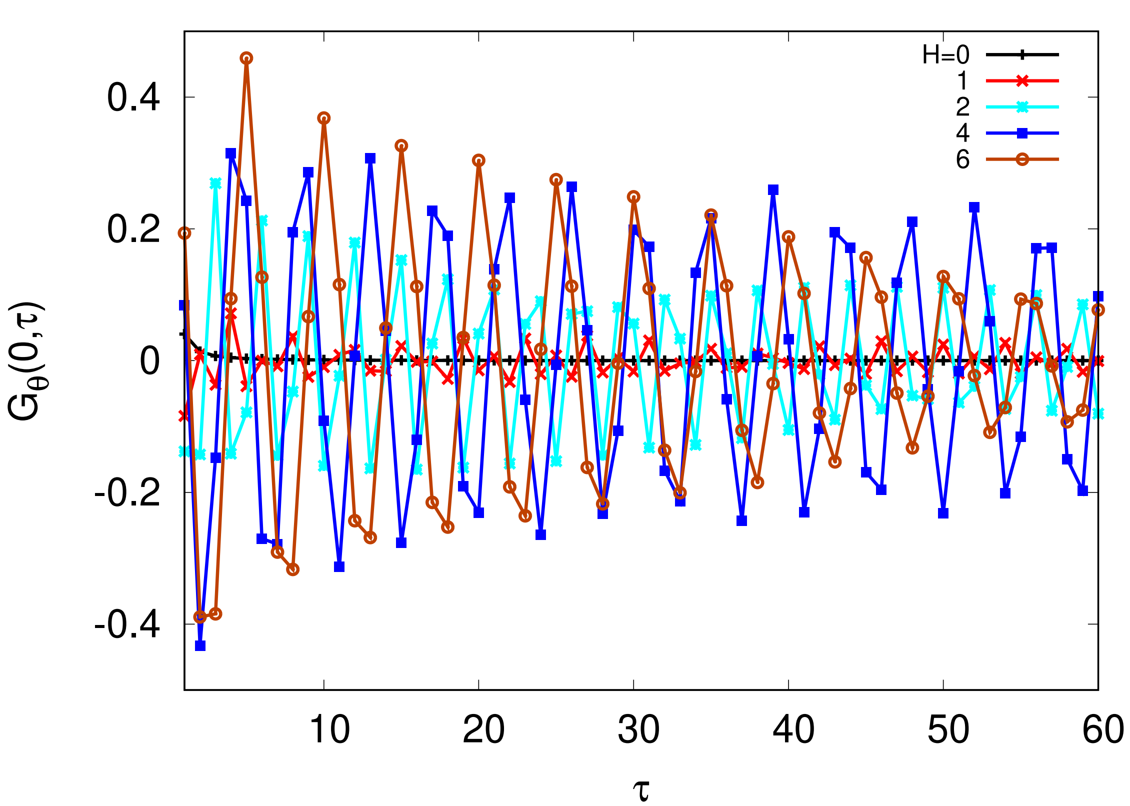
None, except if θ jumps by $2n\pi$ in Im. time.

i.e. only coupling to Topological excitations:
$\hat{\mathbf{z}} \cdot \mathbf{B} \ \sum \ \rho_{\mathbf{w}}(\tau)$

Monte-carlo calculations: Lijun Zhu

Time crystals in imaginary time !





1. Prediction for magnetic flucs. measurable by neutron scattering:

$$\chi"(q,\omega) \propto B/\omega, for\ \omega \gtrsim T, B \gtrsim T$$

2. Prediction for single-particle scattering rate in Fe-based compounds:

Scattering rate ~ ω, nearly ind. of angle - no hot-spots at AFM-QC !

3. In the region in which the specific heat is,

$$C_v/T \propto Log\ T$$

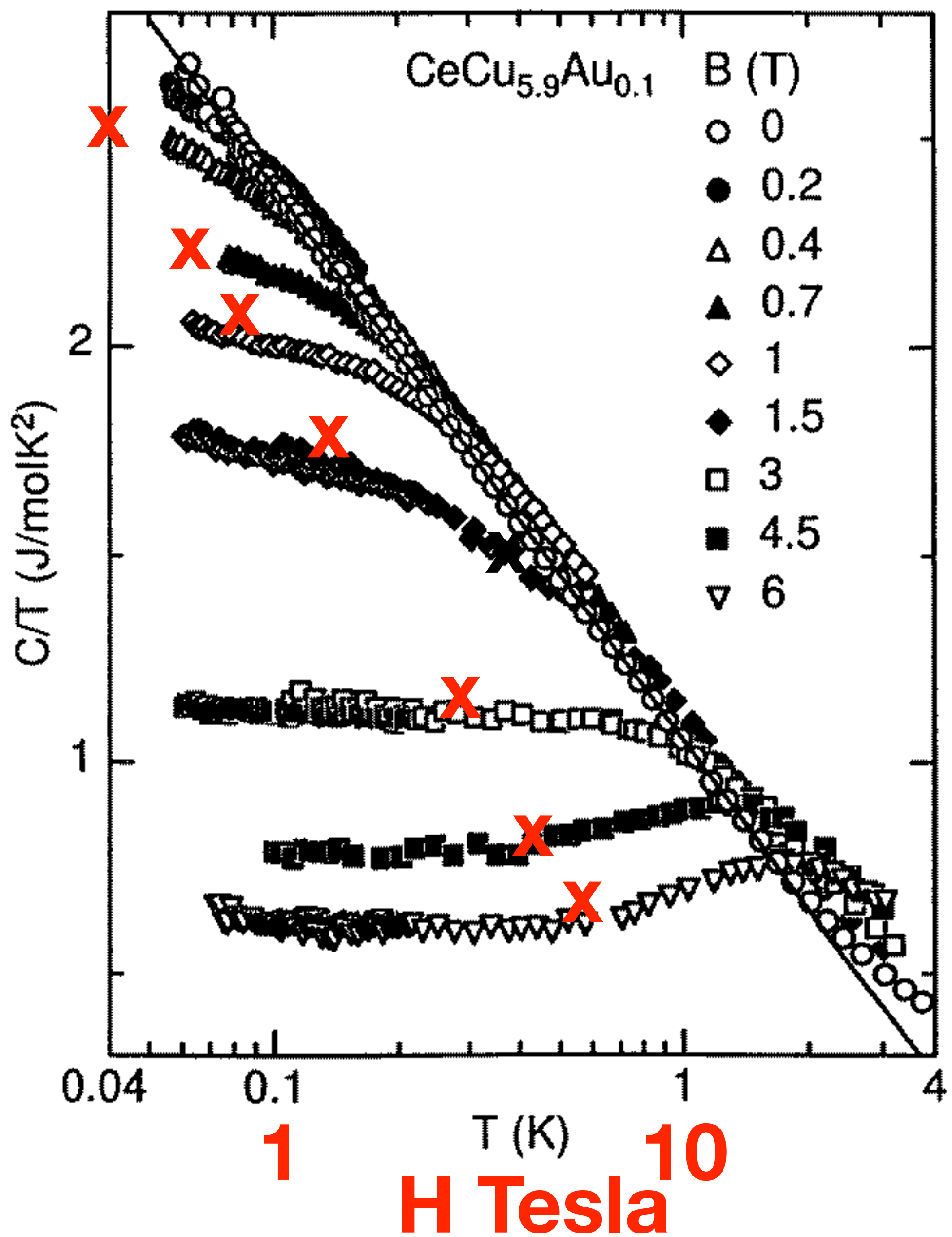
simple calculation shows that it should acquire a contribution

$$C_v/T \propto Log\ B$$

for $B >> T$.

Verified in CeCu(6-x)Au(x) and in CeCoIn(5).

von Lohneysen et al. (1999).

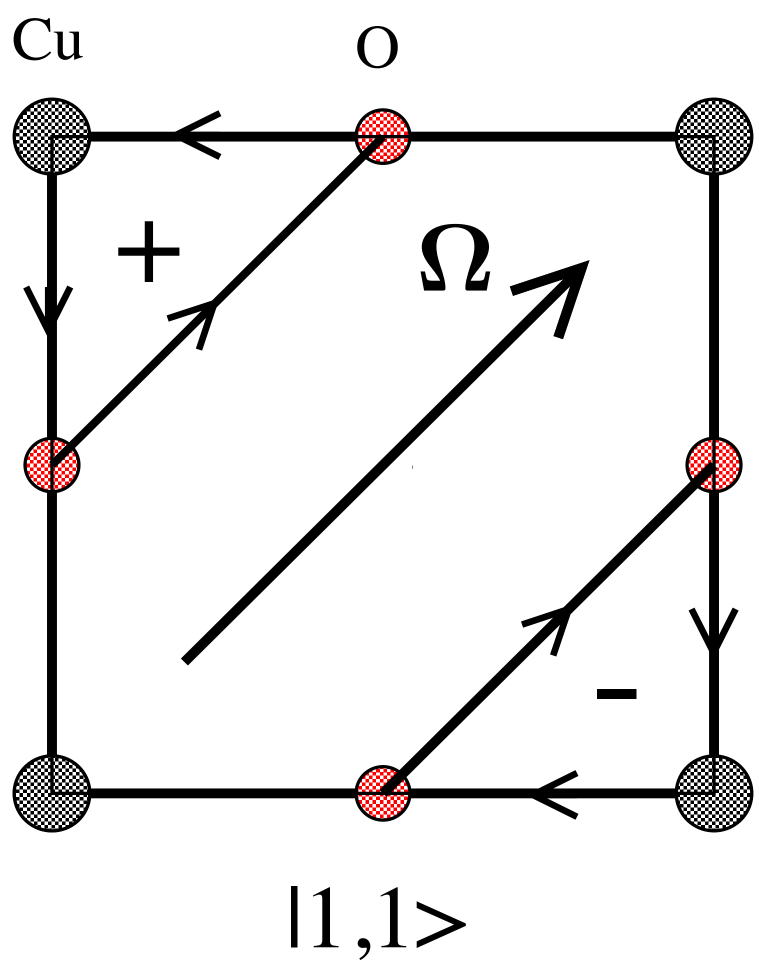


Summary:

The solution of the dissipative quantum xy model reveals a simple and unusual correlation function : Product of a function of space and a function of imaginary time. Freedom of space and time. Only possible with topological excitations.

Quantum-critical thermodynamic and transport properties in cuprates and in antiferromagnetic metals are very well understood by this solution.

For the AFM's some questions of crossover to the xy model remain.



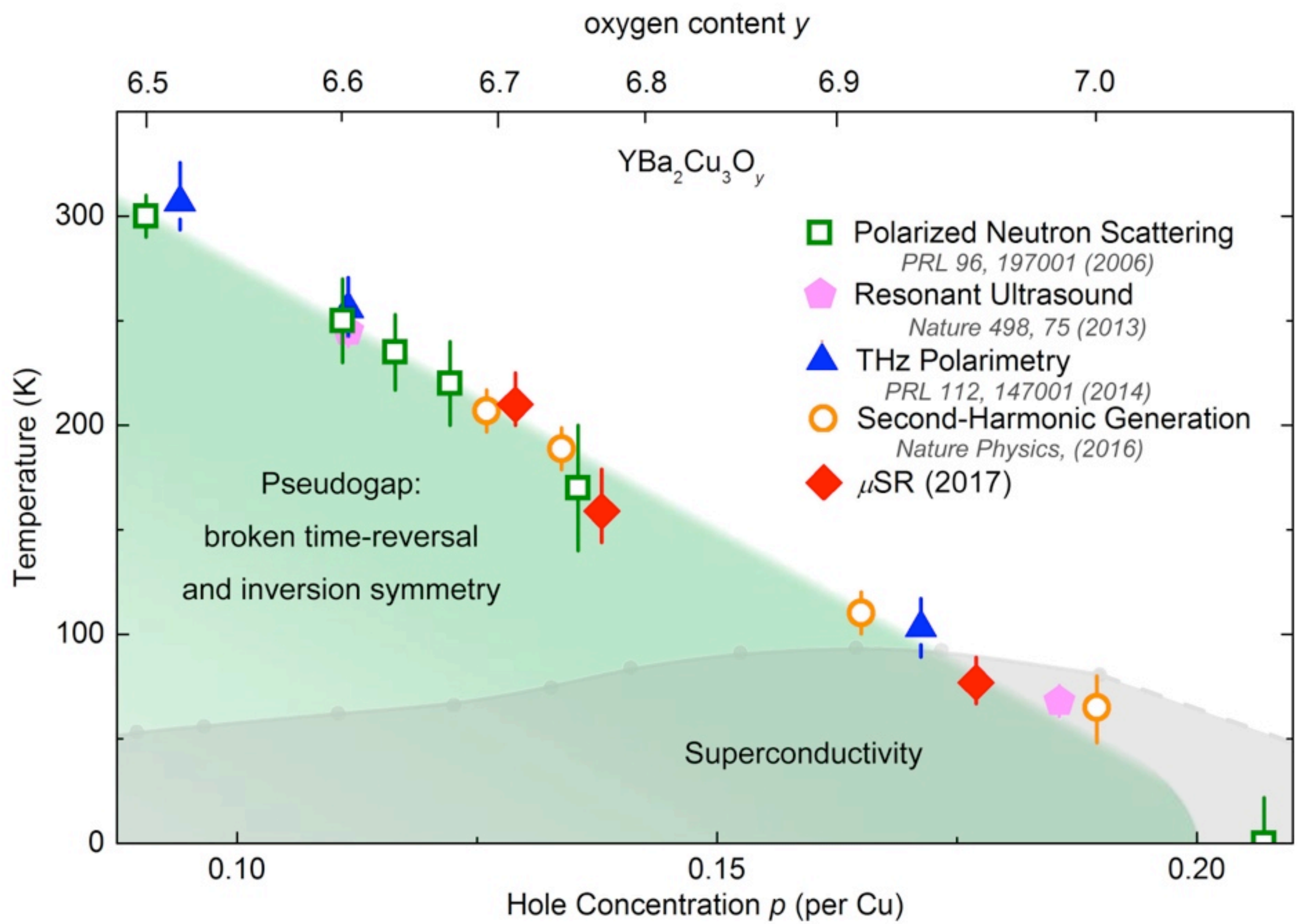
1997: Proposed

Order parameter:

$$\mathbf{\Omega} \equiv \int_{cell} \Big(\mathbf{M}(\mathbf{r}) \times \hat{\mathbf{r}}\Big).$$

Translational Symm. Preserved.

Time-reversal, 4-fold rotation
and all except one reflection broken.
(Magneto-electric)



Polarized neutron scattering in four families of cuprates with the same symmetry discovered.

Also, Dichroic ARPES in BISCCO.

Tribute to Lev Gor’kov

Interactions which shaped important aspects of my scientific work.

1980’s : Volovik and Gor’kov - Classifications of symmetries of superconductors in crystals.
Buried in the results: Triplet superconductors cannot have line-nodes of gap for non-zero SO interactions - The anisotropic superconductors discovered had to be “D-wave - singlets”.

Discussions on how **Fermi-liquid renormalizations** in heavy-fermions are qualitatively different from Fermi-liquid renormalizations in liquid He-3. **z is not an unmentionable!**

2000-2016: Cuprate Physics.

How anisotropic pseudogap might arise in Q=0 ordered state but with domains?

Deciphering effective interactions from experiments when there are no small parameters so that no calculations are reliable? **Or the physics of Irreducible interactions.**

To what extent do “Methods of QFT …” as in AGD’s book (1963) help discover physics beyond quasi-particles and superconductivity in cuprates, heavy fermions, Fe-based compounds, etc. ?

Nature of Irreducible vertices and the validity of the Eliashberg equations.