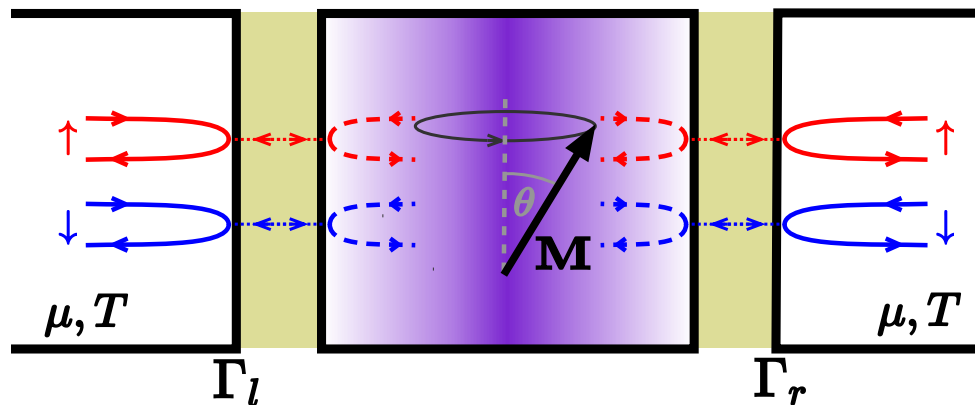
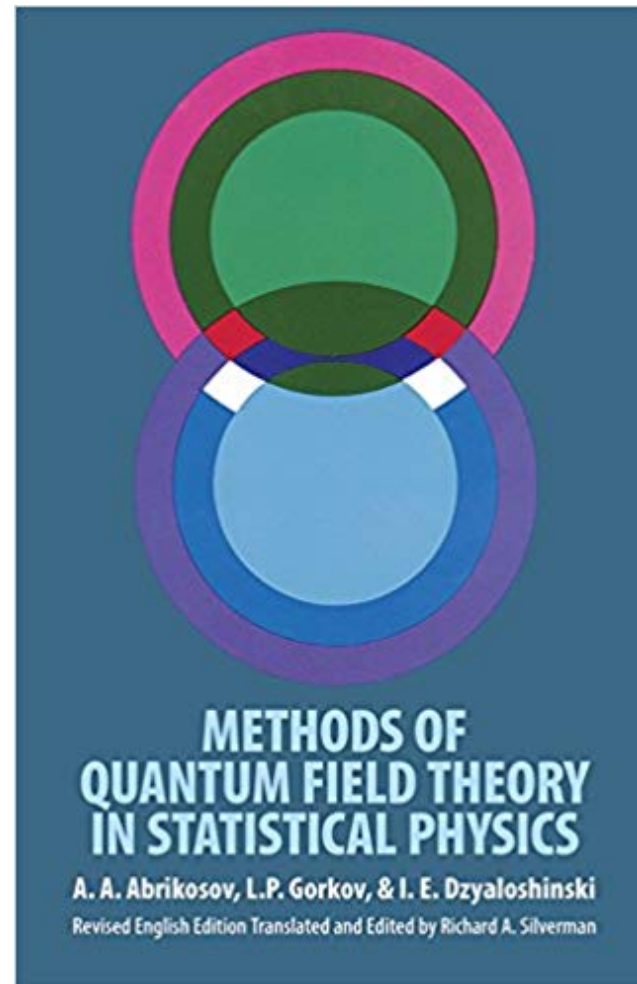


Noise of charge current generated by a driven magnet

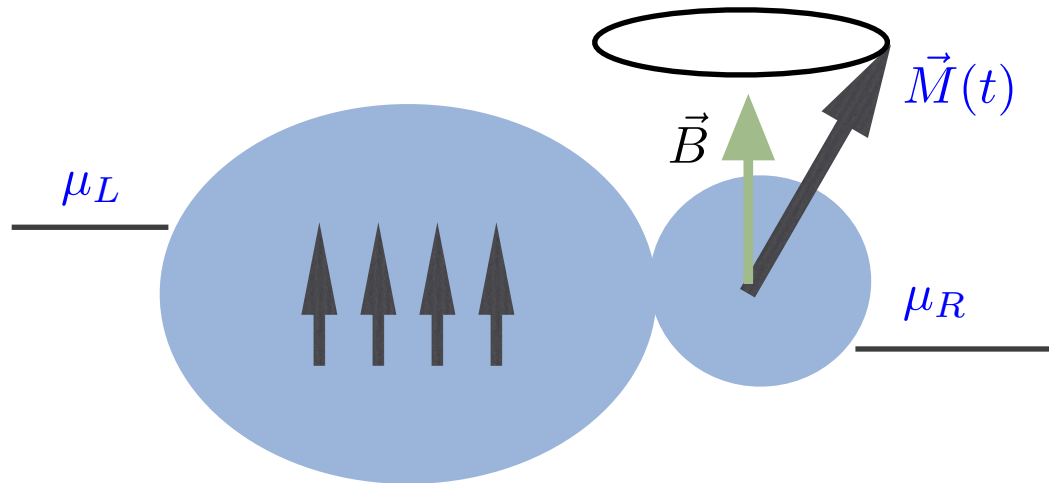
Tim Ludwig, Igor S. Burmistrov, Yuval Gefen, Alexander Shnirman

[arXiv:1906.02730](https://arxiv.org/abs/1906.02730)





Two main (dual) effects: spin transfer torque



$$\frac{d\mathbf{M}}{dt} = -\mathbf{B} \times \mathbf{M} - \alpha(\theta) \frac{\mathbf{M}}{M_0} \times \frac{d\mathbf{M}}{dt} + \mathbf{M} \times \left(\frac{\mathbf{I}_s}{S} \times \frac{\mathbf{M}}{M_0} \right)$$

L. Berger, Phys. Rev. B 54, 9353 (1996)

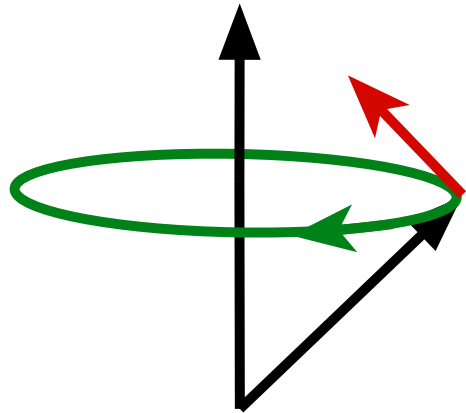
J. C. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996)

Landau & Lifshitz, Phys. Z. Sowietunion 8, 153 (1935)

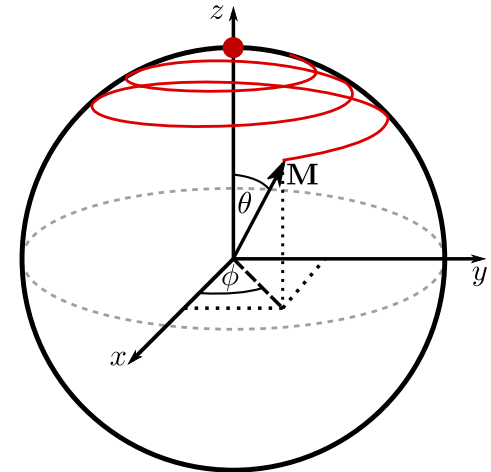
T.L. Gilbert (1955, 2004)

Landau-Lifshitz-Gilbert equation

$$\dot{\mathbf{m}} = \mathbf{m} \times \mathbf{B} - \alpha \mathbf{m} \times \dot{\mathbf{m}}$$



$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix}$$

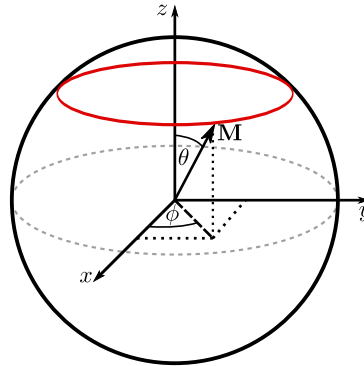


Landau-Lifshitz-Gilbert equation: FMR

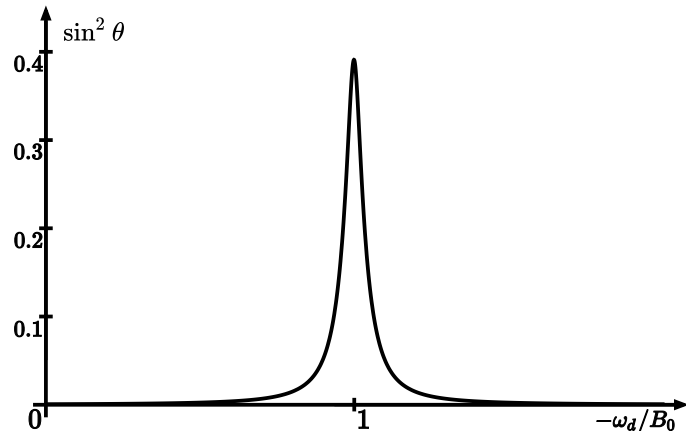
$$\dot{\mathbf{m}} = \mathbf{m} \times \mathbf{B} - \alpha \mathbf{m} \times \dot{\mathbf{m}}$$

$$\mathbf{B} = \begin{pmatrix} \Omega \cos \omega_d t \\ \Omega \sin \omega_d t \\ B_0 \end{pmatrix}$$

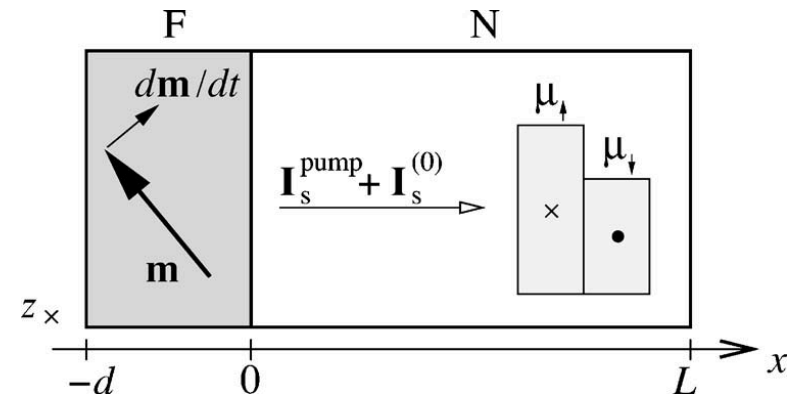
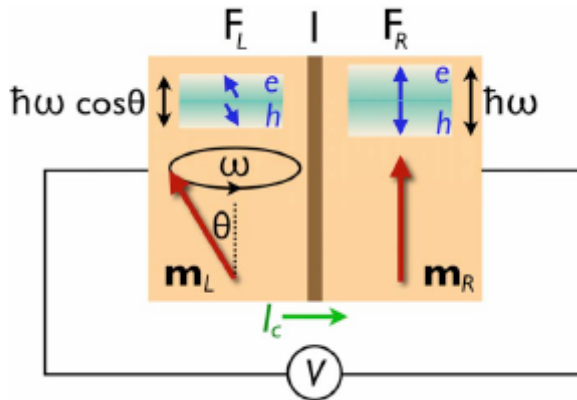
FMR-driven
steady state
precessions



stationary values
of the polar angle θ



Two main (dual) effects: spin/charge pumping

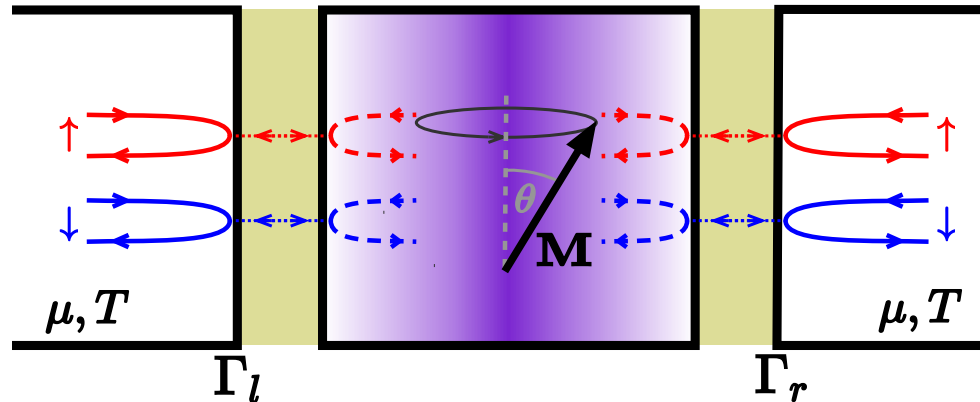


L. Berger, Phys. Rev. B **59**, 11465 (1998)

Y. Tserkovnyak et al., Rev. Mod. Phys. **77**, 1375 (2005)

Y. Tserkovnyak et al., Phys. Rev. B **78**, 020401(R) (2008)

Result: Noise without average current



total conductance

$$g_t = 2(\rho_d^\uparrow + \rho_d^\downarrow) \frac{\Gamma_l \Gamma_r}{\Gamma_l + \Gamma_r}$$

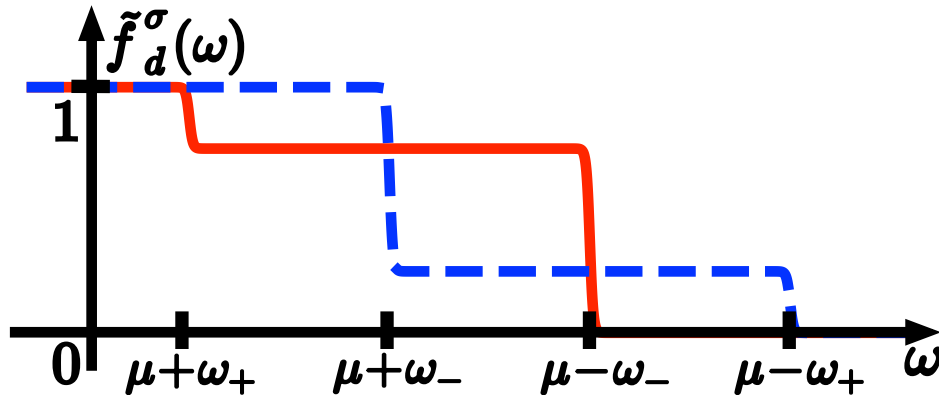
The average charge current vanishes ...

$$I = 0$$

... but its zero-frequency noise remains even at low T

$$S = 2g_t T + g_t \sin^2 \theta \left(\dot{\phi} \coth \frac{\dot{\phi}}{2T} - 2T \right) / 2$$

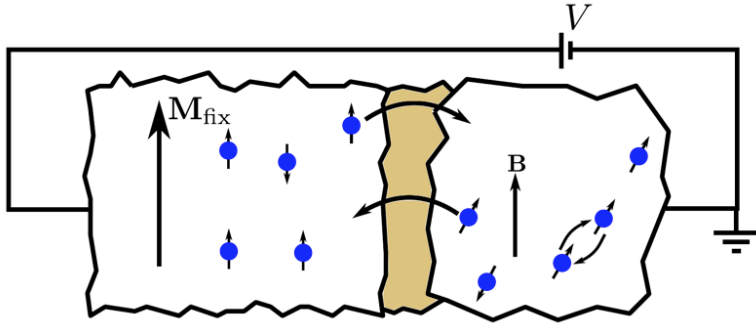
Result: Stationary distribution function



$$\omega_{\pm} = \dot{\phi} \frac{1 \pm \cos \theta}{2}$$

Derivation of Landau-Lifshitz-Gilbert equation

AS, Y. Gefen, A. Saha, I. S. Burmistrov, M. N. Kiselev, A. Altland,
Phys. Rev. Lett. 114, 176806 (2015)



- no anisotropy
- no internal relaxation mechanism
- Stoner ferromagnet:

$$\rho_d J > 1 \quad |\mathbf{M}| \approx M_0$$

The Hamiltonian:

I. L. Kurland, I. L. Aleiner, and B. L. Altshuler,
Phys. Rev. B 62, 14886 (2000)

$$H_{\text{dot}} = \sum_{\alpha\sigma} \epsilon_{\alpha} a_{\alpha\sigma}^{\dagger} a_{\alpha\sigma} - \mathbf{B}\mathbf{S} - JS^2$$

$$H_{\text{lead}} = \sum_{\gamma\sigma} \left(\epsilon_{\gamma} - \sigma \frac{M_{\text{fix}}}{2} + V \right) c_{\gamma\sigma}^{\dagger} c_{\gamma\sigma}$$

$$H_{\text{tun.}} = \sum_{\alpha\gamma\sigma} t_{\alpha\gamma} a_{\alpha\sigma}^{\dagger} c_{\gamma\sigma} + h.c.$$

total spin:

$$\mathbf{S} = \frac{1}{2} \sum_{\alpha\sigma\sigma'} a_{\alpha\sigma}^{\dagger} \vec{\sigma}_{\sigma\sigma'} a_{\alpha\sigma'}$$

The electron distribution:

$$n_d(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$$

$$n_l(\epsilon) = \frac{1}{e^{\beta(\epsilon-(\mu+V))} + 1}$$

Derivation of Landau-Lifshitz-Gilbert equation

driving \longrightarrow **non-equilibrium** \longrightarrow **Keldysh formalism**

$$i\mathcal{S}[\bar{\Psi}, \Psi] = i \oint_K dt [\bar{\Psi}(i\partial_t)\Psi - H(\bar{\Psi}, \Psi)]$$

decoupling of exchange interaction

$$e^{iJ\mathbf{S}^2} = \int d^3\boldsymbol{\Phi} e^{-i\boldsymbol{\Phi}\cdot\mathbf{S}} e^{-i\frac{|\boldsymbol{\Phi}|^2}{4J}}$$

Integrating out fermions

Effective action


$$i\mathcal{S}[\mathbf{M}] = \text{tr} \ln \left[-i \left(G_{d,0}^{-1} + \mathbf{M} \frac{\boldsymbol{\sigma}}{2} - \boldsymbol{\Sigma} \right) \right] - i \overbrace{\int_K dt \frac{(\mathbf{M} - \mathbf{B})^2}{4J}}^{-\mathcal{S}_B}$$

Broadening by the lead

$$\mathbf{M} \equiv \boldsymbol{\Phi} + \mathbf{B}$$

$$\boldsymbol{\Sigma} = t_l G_l t_l^\dagger$$

Gauge transformation

$$i\mathcal{S} = \text{tr} \ln \left[-i \left(G_{d,0}^{-1} + \mathbf{M} \frac{\boldsymbol{\sigma}}{2} - \Sigma \right) \right] + i\mathcal{S}_B$$


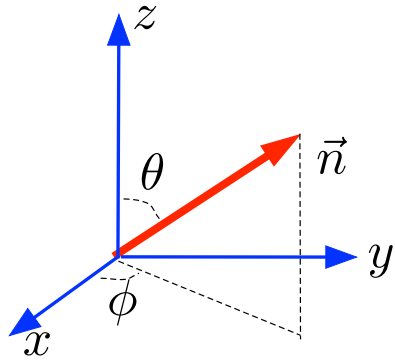
$$\mathbf{M}(t) = M(t) \vec{n}(t) \approx M \vec{n}(t)$$

$$\vec{n} \cdot \vec{\sigma} = R \sigma_z R^\dagger \quad \text{SU(2) gauge transformation}$$

$$Q(t) \equiv R^\dagger (-i\partial_t) R \quad \text{geometric vector potential (connection)}$$

$$i\mathcal{S} = \text{tr} \ln \left[-i \left(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2} \sigma_z}_{G_{d,z}^{-1}} - Q - R^\dagger \Sigma R \right) \right] + i\mathcal{S}_B$$

SU(2) vector potential



$$\vec{n} \cdot \vec{S} = R S_z R^\dagger$$

$$R \in SU(2)/U(1)$$

$$R = \exp \left[-\frac{i\phi}{2} \sigma_z \right] \exp \left[-\frac{i\theta}{2} \sigma_y \right] \exp \left[\frac{i(\phi - \chi)}{2} \sigma_z \right]$$

$$Q \equiv R^\dagger (-i\partial_t) R = Q_{\parallel} + Q_{\perp}$$

$$Q_{\parallel} \equiv \frac{1}{2} \left[\dot{\phi}(1 - \cos \theta) - \dot{\chi} \right] \sigma_z \quad \text{Berry's phase, gauge dependent}$$

$$Q_{\perp} \equiv -\frac{1}{2} \left[\dot{\theta} \sigma_y - \dot{\phi} \sin \theta \sigma_x \right] \exp [i(\phi - \chi) \sigma_z]$$

Landau-Zener,
neglected: adiabatic approximation

Tunneling expansion, AES

$$i\mathcal{S}_M = \text{tr} \ln \left[-i \left(G_{d,z}^{-1} - Q_q - R^\dagger \Sigma R \right) \right] \overbrace{-i \oint_K dt \frac{|\vec{\mathbf{M}} - \vec{\mathbf{B}}|^2}{4J}}^{i\mathcal{S}_B}$$

Expansion

first order in $Q_q \longrightarrow i\mathcal{S}_{WZ\text{NW}} = -\text{tr} [G_{d,z} Q_q] = iS \oint \dot{\phi} (1 - \cos \theta)$

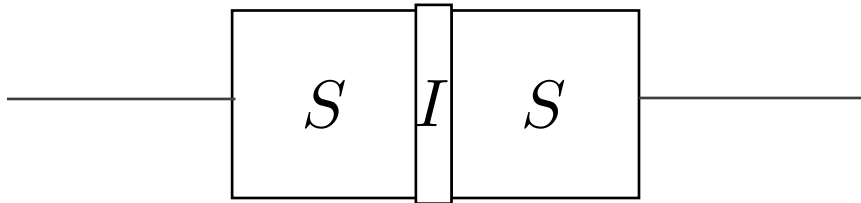
first order in $\Sigma \longrightarrow i\mathcal{S}_{AES} = -\text{tr} [G_{d,z} R^\dagger \Sigma R]$

in original U(1) AES

$$R^\dagger(t) R(t') \rightarrow e^{i\psi(t)} e^{-i\psi(t')}$$

V. Ambegaokar, U. Eckern, G. Schön
Phys. Rev. Lett. 48, 1745-1748 (1982)

AES action for Josephson contact



$$\begin{aligned}
 i\mathcal{S}_{AES} = & i \int dt \frac{C\dot{\phi}^2}{2e^2} - \int dt_1 dt_2 \alpha(t_1, t_2) \cos [\phi(t_1) - \phi(t_2)] \\
 & - \int dt_1 dt_2 \beta(t_1, t_2) \cos [\phi(t_1) + \phi(t_2)]
 \end{aligned}$$

V. Ambegaokar, U. Eckern, G. Schön
 Phys. Rev. Lett. **48**, 1745-1748 (1982)

Explicit form for non-magnetic lead

$$i\mathcal{S}_M^{AES} = - \int dt_1 dt_2 \alpha(t_1 - t_2) \text{tr} [R(t_1)R^{-1}(t_2)]$$

Matsubara

$$\alpha(\tau) = \frac{\pi g}{\sin^2(\pi\tau/\beta)}$$

Tunneling conductance

$$g = \pi \rho_{lead} \rho_{dot} |T|^2$$

$$\text{tr} [R(t_1)R^{-1}(t_2)] =$$

$$\cos \frac{\theta(t_1)}{2} \cos \frac{\theta(t_2)}{2} \cos \left(\frac{\chi(t_1) - \chi(t_2)}{2} \right)$$

$$+ \sin \frac{\theta(t_1)}{2} \sin \frac{\theta(t_2)}{2} \cos \left(\phi(t_1) - \phi(t_2) - \frac{\chi(t_1) - \chi(t_2)}{2} \right)$$

Gauge invariance ???

Variation of the action \longrightarrow EOM

AS, Y. Gefen, A. Saha,
I. S. Burmistrov, M. N. Kiselev, A. Altland
[Phys. Rev. Lett. 114, 176806 \(2015\)](#)

$$i\mathcal{S} = i\mathcal{S}_{\text{WZNW}} + i\mathcal{S}_B + i\mathcal{S}_{\text{AES}}$$

$$\frac{d\mathbf{M}}{dt} = -\mathbf{B} \times \mathbf{M} - \alpha(\theta) \frac{\mathbf{M}}{M_0} \times \frac{d\mathbf{M}}{dt} + \mathbf{M} \times \left(\frac{\mathbf{I}_s}{S} \times \frac{\mathbf{M}}{M_0} \right)$$

Gilbert-damping [1]:

$$\alpha(\theta) = \frac{\tilde{g}(\theta)}{S}$$

Spin-torque current [2]:

$$I_s = g_s V$$

Charge current [3]:

$$I = 4g(\theta)V - g_s \sin^2 \theta \dot{\phi}$$

[1] A. L. Chudnovskiy, et al. PRL 101 066601 (2008).

[2] J. C. Slonczewski, JMMM 159, L1 (1996). & L. Berger, Phys. Rev. B 54, 9353 (1996)

[3] L. Berger, Phys. Rev. B 59, 11465 (1998), Y. Tserkovnyak et al., Phys. Rev. B 78, 020401(R) (2008)

Technical details

$$i\mathcal{S} = \text{tr} \ln \left[-i \left(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2} \sigma_z}_{G_{d,z}^{-1}} - Q - R^\dagger \Sigma R \right) \right] + i\mathcal{S}_B$$

$$G_{d,z}(\epsilon) = \begin{pmatrix} -2\pi i \delta(\epsilon - \epsilon_\alpha + \frac{M_0}{2} \sigma_z) F_d(\epsilon) & \frac{1}{\epsilon - \epsilon_\alpha + \frac{M_0}{2} \sigma_z + i0} \\ \frac{1}{\epsilon - \epsilon_\alpha + \frac{M_0}{2} \sigma_z - i0} & 0 \end{pmatrix}$$

$$\Sigma_\sigma(\epsilon) \approx \begin{pmatrix} 0 & i\Gamma_l^\sigma \\ -i\Gamma_l^\sigma & -2i\Gamma_l^\sigma F_l(\epsilon) \end{pmatrix}$$

$F(\epsilon) \equiv 1 - 2n(\epsilon)$
distribution func.

Γ_l^σ spin-resolved level width

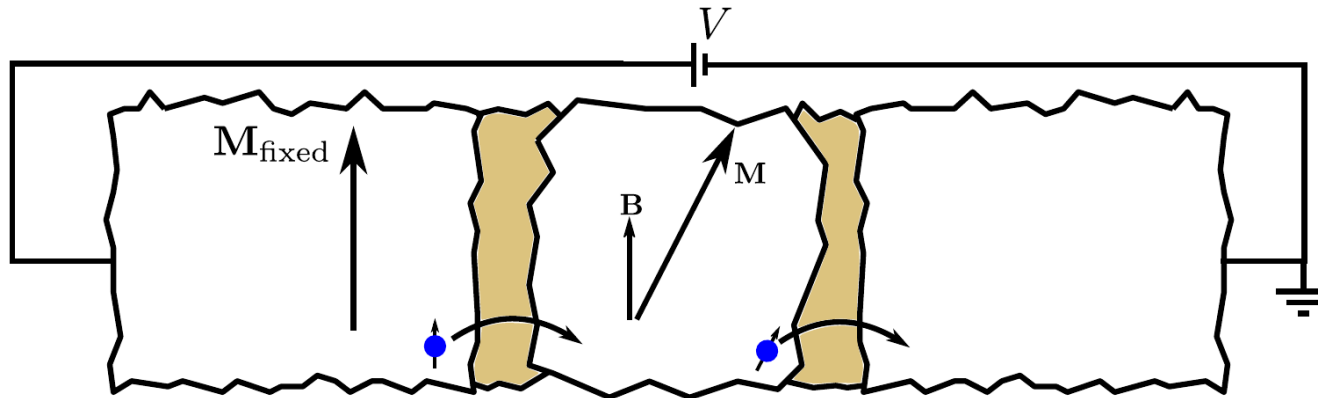
Non-equilibrium taken seriously:

Tim Ludwig, Igor S. Burmistrov, Yuval Gefen, and AS

[Phys. Rev. B 95, 075425 \(2017\).](#)

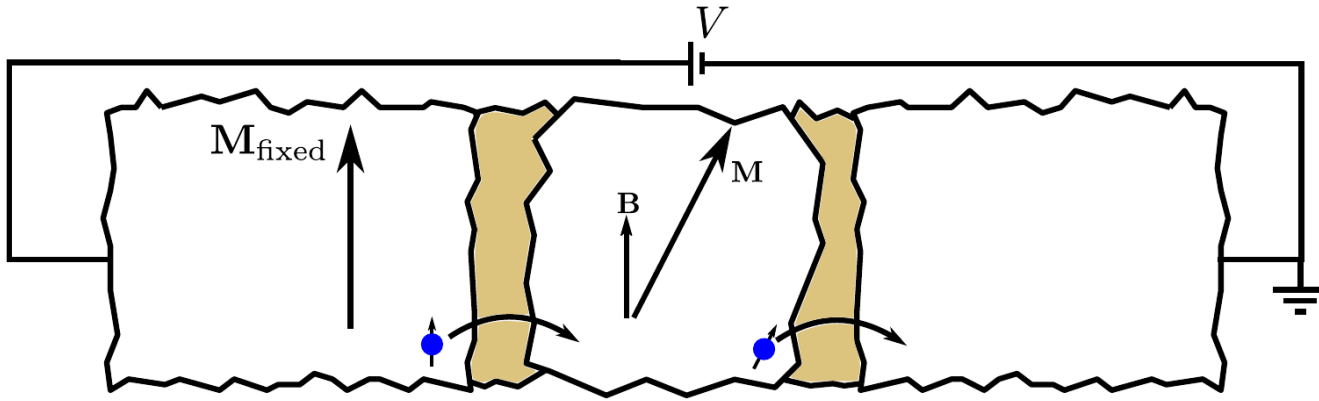
Tim Ludwig, Igor S. Burmistrov, Yuval Gefen, AS

[Phys. Rev. B 99, 045429 \(2019\).](#)



**The electron distribution on the dot
strongly affected by driving!**

Results change drastically



$$i\mathcal{S}[\mathbf{M}, V_d] = \text{tr} \ln \left[-i \left(G_{d,0}^{-1} + \mathbf{M} \frac{\boldsymbol{\sigma}}{2} - V_d - \Sigma \right) \right]$$

$$= \underbrace{-i \int_K dt \frac{(\mathbf{M} - \mathbf{B})^2}{4J}}_{-\mathcal{S}_B} + \underbrace{i \int_K dt \left(\frac{C V_d^2}{2} + V_d N_0 \right)}_{\mathcal{S}_C}$$

$$\Sigma = \Sigma_l + \Sigma_r = t_l G_l t_l^\dagger + t_r G_r t_r^\dagger$$

AES-strategy:

$$i\mathcal{S} = \text{tr} \ln \left[-i \left(G_{d,0}^{-1} + \underbrace{\mathbf{M} \frac{\sigma}{2}}_{SU(2)} - V_d - \Sigma \right) \right] + i\mathcal{S}_B + i\mathcal{S}_C$$

(1) shift the time dependent fields to the tunneling part (gauge trafo):

$$U = \underbrace{e^{-i\frac{\phi}{2}\sigma_z} e^{-i\frac{\theta}{2}\sigma_y} e^{i\frac{\phi-\chi}{2}\sigma_z}}_{SU(2)} e^{-i\psi} \quad \dot{\psi} = V_d$$

(2) expand in the self-energy (tunneling) (and in the Berry-phase):

$$i\mathcal{S} = \text{tr} \ln \left[-i \left(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2}\sigma_z}_{G_{d,z}^{-1}} - Q_q - U^\dagger \Sigma U \right) \right] + i\mathcal{S}_B + i\mathcal{S}_C$$

AES-strategy:

$$i\mathcal{S} = \text{tr} \ln \left[-i \left(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2} \sigma_z}_{G_{d,z}^{-1}} - Q_q - U^\dagger \Sigma U \right) \right] + i\mathcal{S}_B + i\mathcal{S}_C$$

$$i\mathcal{S}_{\text{AES}} = -\text{tr} [G_{d,z} U^\dagger \Sigma U]$$

$$i\mathcal{S}_{\text{WZNW}} = -\text{tr} [G_{d,z} Q_q]$$

We know the self-energy

$$\Sigma_\sigma(\epsilon) \approx \begin{pmatrix} 0 & i(\Gamma_l^\sigma + \Gamma_r) \\ -i(\Gamma_l^\sigma + \Gamma_r) & -2i[\Gamma_l^\sigma F_l(\epsilon) + \Gamma_r F_r(\epsilon)] \end{pmatrix}$$

Which distribution should we use for the dot?
(kinetic equation)


But what about the GF?

$$G_{d,z}^{-1}(\epsilon) = \begin{pmatrix} 0 & \epsilon - \epsilon_\alpha + \frac{M_0}{2} \sigma_z - i0 \\ \epsilon - \epsilon_\alpha + \frac{M_0}{2} \sigma_z + i0 & -2i0 F_?(\epsilon) \end{pmatrix}$$

“improved” AES-like-strategy:

expand around a “classical” trajectory

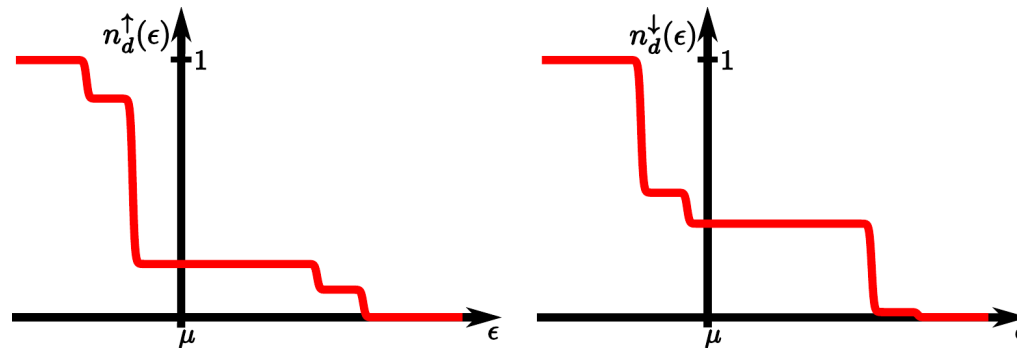
$$i\mathcal{S} = \text{tr} \ln \left[-i \left(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2} \sigma_z}_{G_d^{-1}} - U_0^\dagger \Sigma U_0 - Q_q - (U^\dagger \Sigma U - U_0^\dagger \Sigma U_0) \right) \right] + i\mathcal{S}_B + i\mathcal{S}_C$$

Kinetic equation  **Distribution function**

$$i\mathcal{S}_{\text{AES}} = -\text{tr} \left[G_d (U^\dagger \Sigma U - U_0^\dagger \Sigma U_0) \right]$$

“improved” AES-like-strategy: multi-step distribution function

$$\begin{aligned}
 F_d^\sigma(\epsilon) = & \frac{1}{\Gamma_\sigma(\theta_0)} \left[\cos^2 \frac{\theta_0}{2} \Gamma_l^\sigma F(\epsilon - \sigma B_- + V_d^0 - V) \right. \\
 & + \sin^2 \frac{\theta_0}{2} \Gamma_l^{\bar{\sigma}} F(\epsilon - \bar{\sigma} B_+ + V_d^0 - V) \\
 & + \cos^2 \frac{\theta_0}{2} \Gamma_r F(\epsilon - \sigma B_- + V_d^0) \\
 & \left. + \sin^2 \frac{\theta_0}{2} \Gamma_r F(\epsilon - \bar{\sigma} B_+ + V_d^0) \right]
 \end{aligned}$$



$$B_\pm \equiv B_0(1 \pm \cos \theta_0)/2$$

AES-like-action



Landau-Lifshitz-Gilbert-Slonczewski equation:

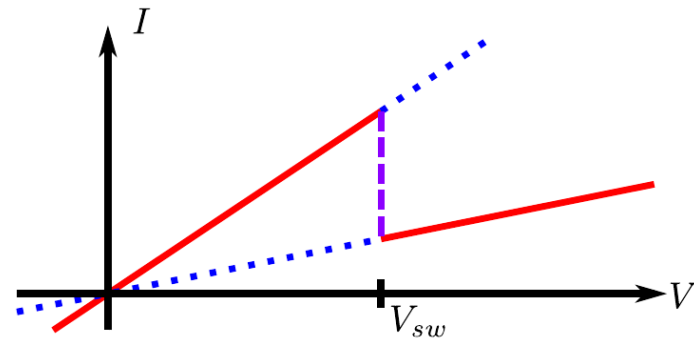
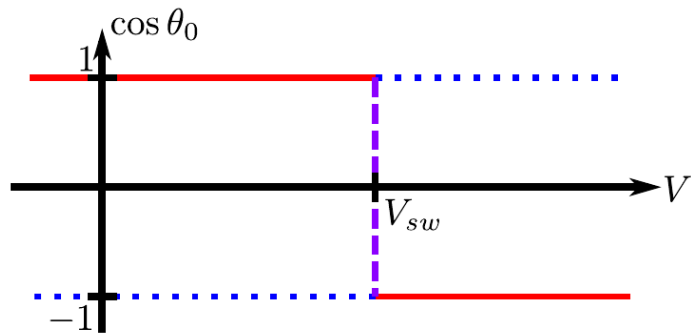
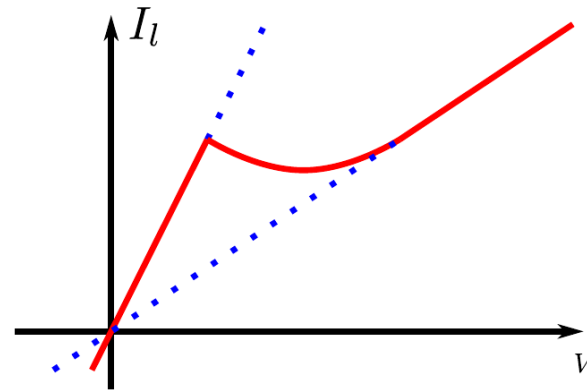
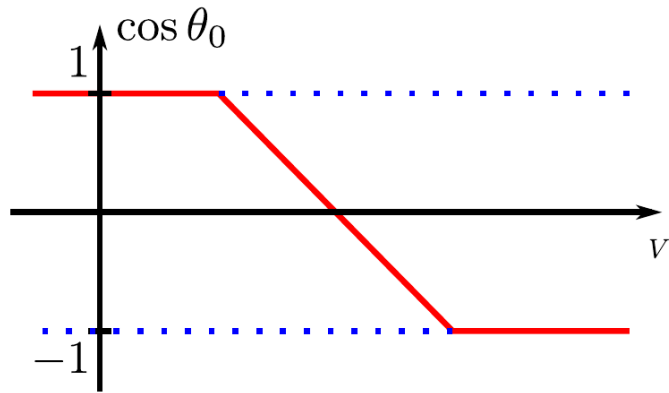
$$\frac{d\mathbf{M}}{dt} = -\mathbf{B} \times \mathbf{M} - \alpha(\theta) \frac{\mathbf{M}}{M_0} \times \frac{d\mathbf{M}}{dt} + \mathbf{M} \times \left(\frac{\mathbf{I}_s}{S} \times \frac{\mathbf{M}}{M_0} \right)$$

Kirchhoff's law:

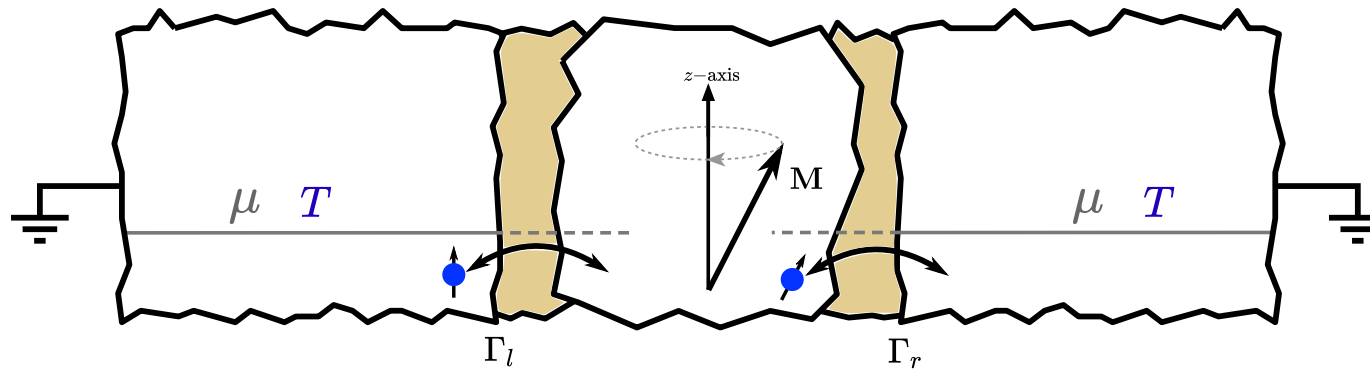
$$C\dot{V}_d = I_l - I_r$$

BUT: I_s, I_l, I_r **Modified completely**

I_s, I_l, I_r Modified completely



FMR driven micromagnet



Counting field

$$i\mathcal{S}(\lambda) = \text{tr} \ln \underbrace{\left[i\partial_t - h_d - \Sigma(\lambda) \right]}_{G_\lambda^{-1}}$$

$$h_d = \epsilon_\alpha - \mathbf{M}(t) \frac{\sigma}{2}$$

this corresponds to: $t_l \rightarrow t_l e^{-i\lambda(t)}$

$$t_l^\dagger \rightarrow t_l^\dagger e^{i\lambda(t)}$$

$$\langle Q \rangle = i\partial_\lambda \mathcal{Z}(\lambda) \Big|_{\lambda=0}$$

$$\langle Q^2 \rangle = (i\partial_\lambda)^2 \mathcal{Z}(\lambda) \Big|_{\lambda=0}$$

It follows:

$$\langle Q \rangle = -i \langle \text{tr}[G_0 \Sigma'] \rangle_{\mathbf{M}(t)}$$

$$\langle\langle Q^2 \rangle\rangle = \langle \text{tr}[G_0 \Sigma''] + \text{tr}[G'_0 \Sigma'] \rangle_{\mathbf{M}(t)}$$

Solution: Rotation in spin-space

$$i\mathcal{S}(\lambda) = \text{tr} \ln \underbrace{\left[i\partial_t - \tilde{h}_d - \tilde{\Sigma}(\lambda) \right]}_{\tilde{G}_\lambda^{-1}}$$

$$h_d = \epsilon_\alpha - \mathbf{M}(t) \frac{\boldsymbol{\sigma}}{2}$$

rotates the **Hamiltonian** ...

$$R^\dagger \mathbf{M} \boldsymbol{\sigma} R = M \sigma_z$$

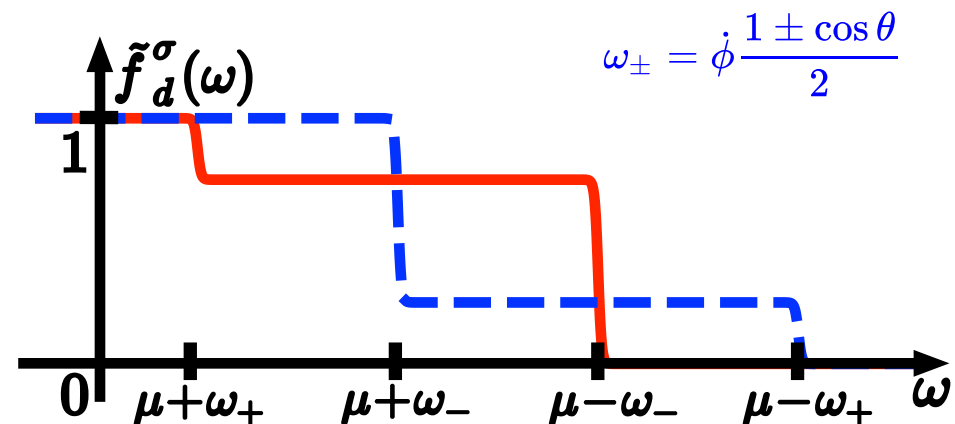
$$\tilde{h}_d = \epsilon_\alpha - M \frac{\sigma_z}{2} - iR^\dagger \dot{R}$$

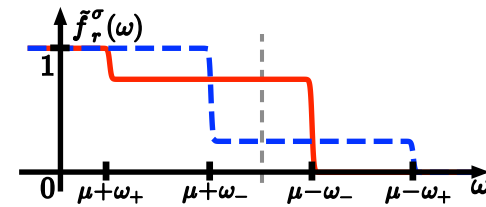
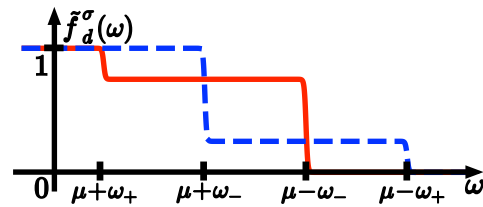
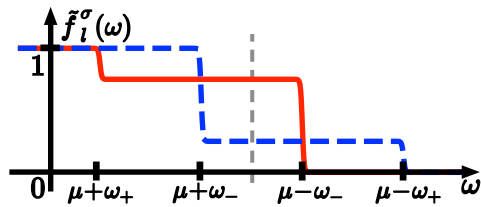
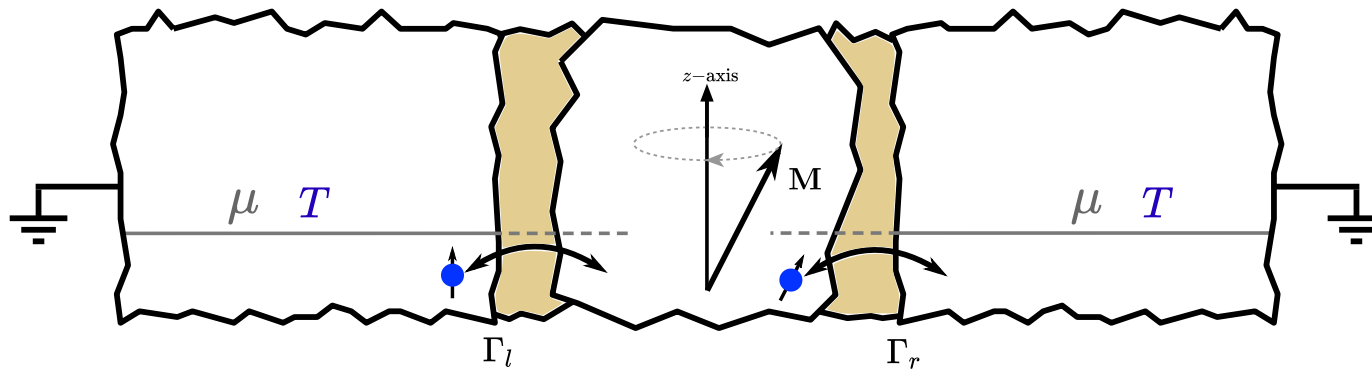
... and the **self-energy** $\tilde{\Sigma} = R^\dagger \Sigma R$

distribution function

$$F_{l/r}^\sigma(\omega) = \cos^2 \frac{\theta}{2} F_{l/r}(\omega + \sigma\omega_-) + \sin^2 \frac{\theta}{2} F_{l/r}(\omega + \bar{\sigma}\omega_+)$$

$$\tilde{F}_d^\sigma(\omega) = [\Gamma_l \tilde{F}_l^\sigma(\omega) + \Gamma_r \tilde{F}_r^\sigma(\omega)] / \Gamma_\Sigma$$

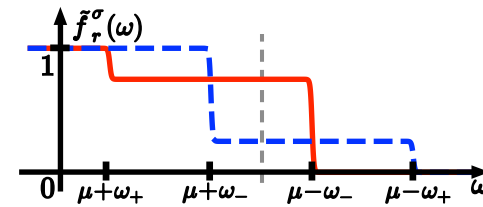
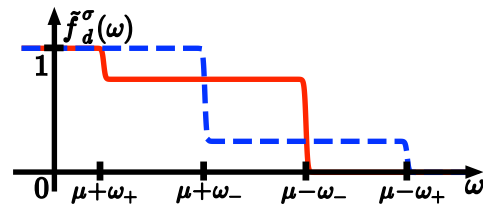
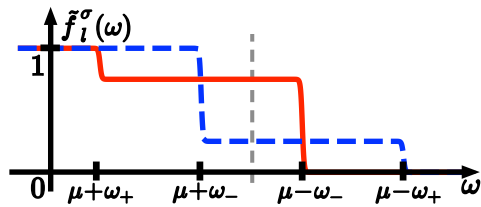
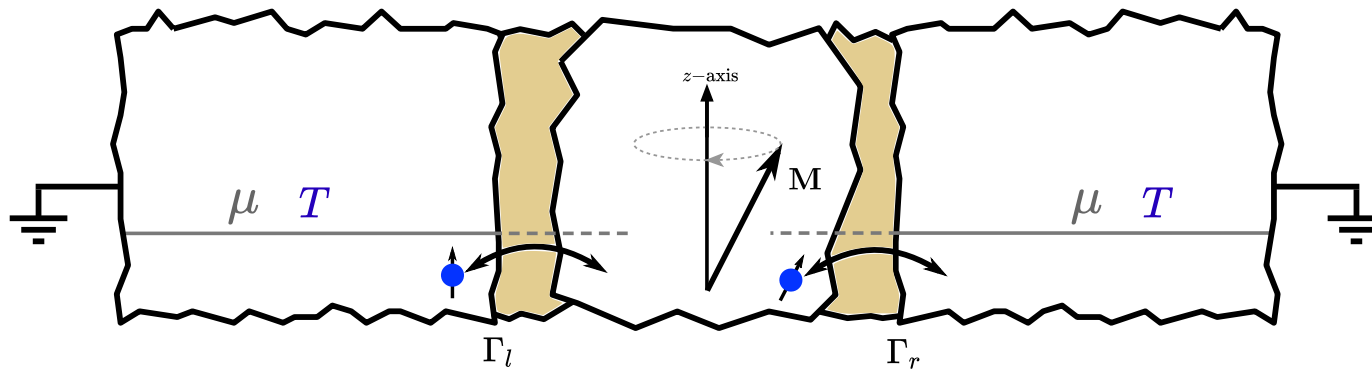




$$\langle Q \rangle = -i \text{tr}[\tilde{G}_0 \tilde{\Sigma}']$$

⇒ **Landauer-formula in rotating-frame**

$$I_l = \sum_{\sigma} \rho_d^{\sigma} \Gamma_l \int d\omega [\tilde{F}_l^{\sigma}(\omega) - \tilde{F}_d^{\sigma}(\omega)] = 0$$



$$\langle\langle Q^2 \rangle\rangle = \text{tr}[\tilde{G}_0 \tilde{\Sigma}'] + \text{tr}[\tilde{G}'_0 \tilde{\Sigma}']$$

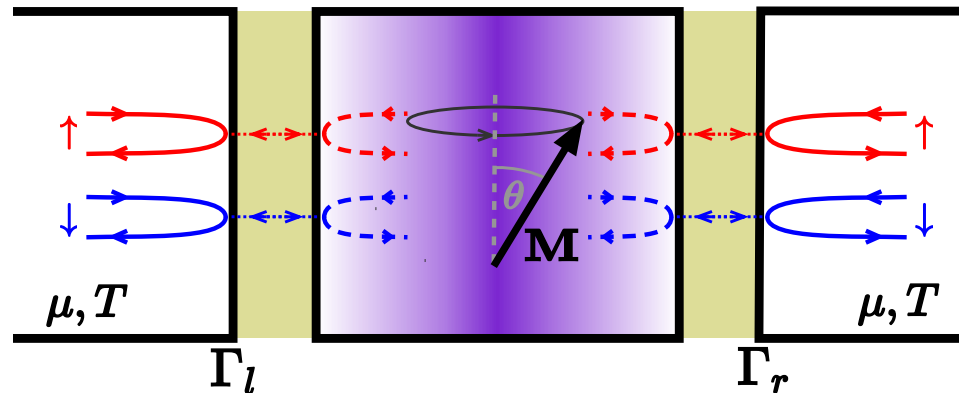
⇒ **zero-frequency noise**

$$g_\sigma = 2\rho_d^\sigma \Gamma_l \Gamma_r / (\Gamma_l + \Gamma_r)$$

$$S_l = \sum_\sigma g_\sigma \int d\omega \left\{ [1 - \tilde{F}_s^\sigma(\omega) \tilde{F}_l^\sigma(\omega)] + \frac{\Gamma_l}{\Gamma_r} \tilde{F}_s^\sigma(\omega) [\tilde{F}_l^\sigma(\omega) - \tilde{F}_s^\sigma(\omega)] \right\}$$

$$S_l = 2g_t T + g_t \sin^2 \theta \left(\dot{\phi} \coth \frac{\dot{\phi}}{2T} - 2T \right) / 2$$

Noise of charge current: even **without** average charge current



$$I = 0$$

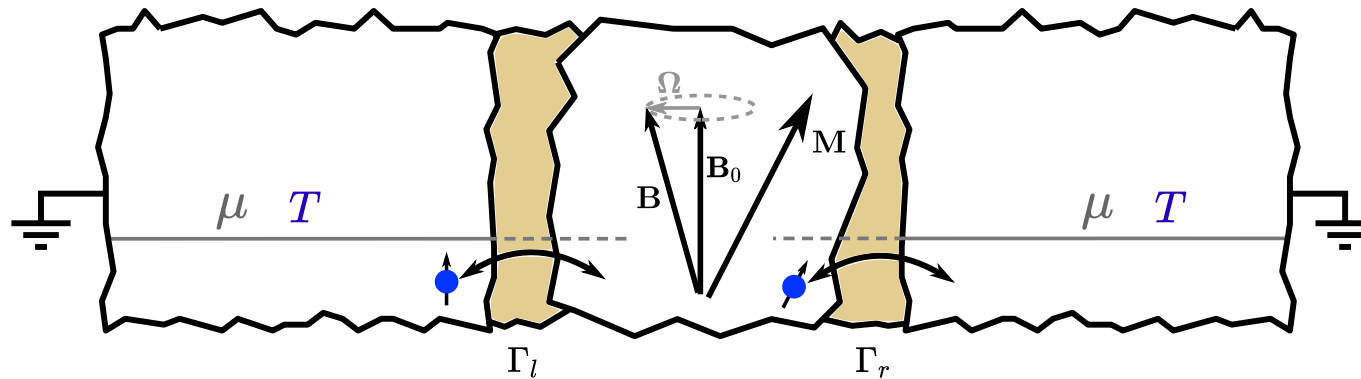
$$S = 2g_t T + g_t \sin^2 \theta \left(\dot{\phi} \coth \frac{\phi}{2T} - 2T \right) / 2$$

high T
↓

$$S = 2g_t T$$

low T
↓

$$S = g_t \sin^2 \theta |\dot{\phi}| / 2$$

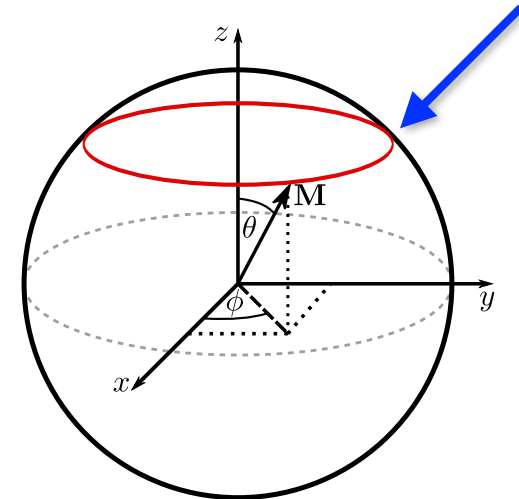
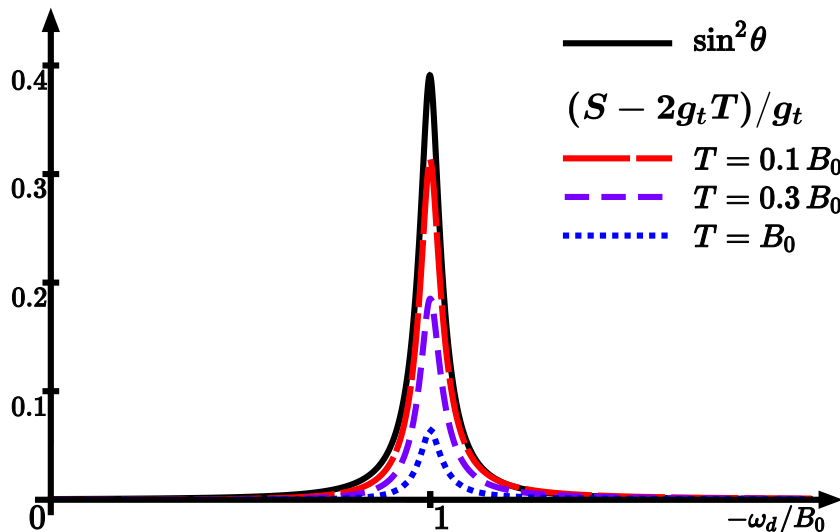


Noise of charge current

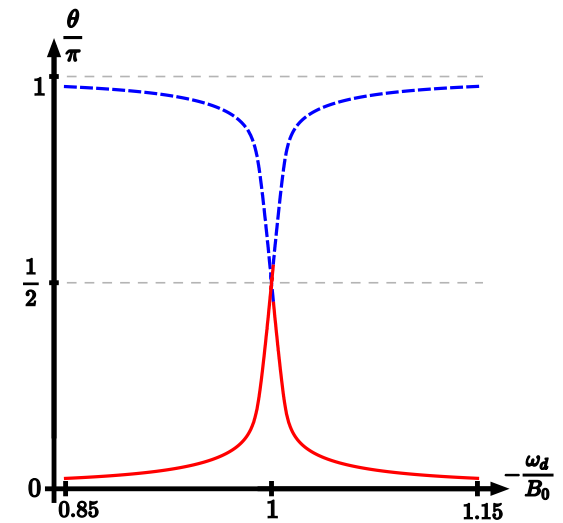
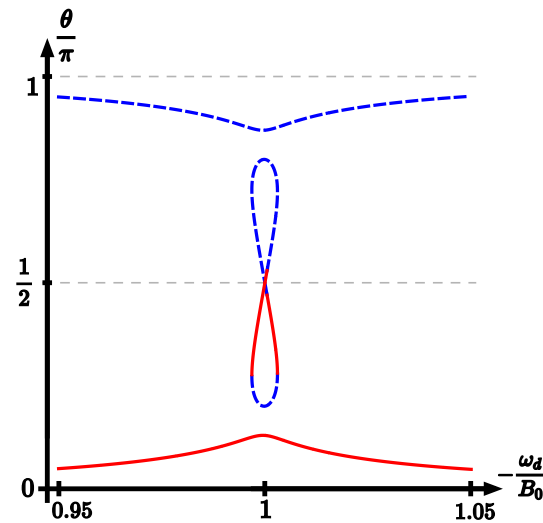
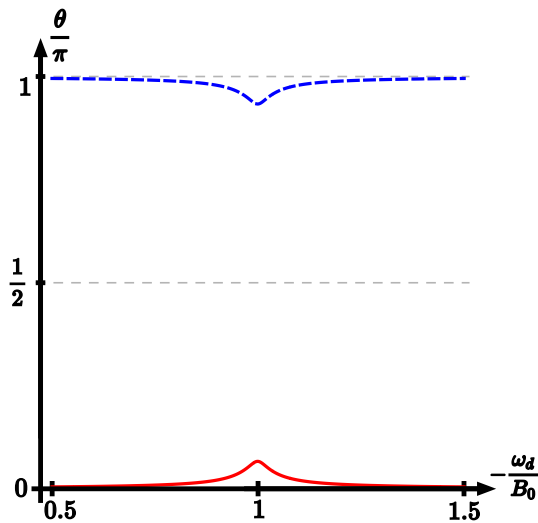
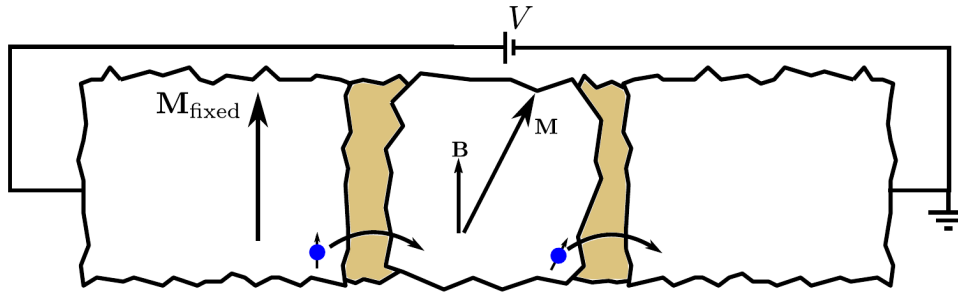
$$S = 2g_t T + g_t \sin^2 \theta \left(\dot{\phi} \coth \frac{\dot{\phi}}{2T} - 2T \right) / 2$$

$$\mathbf{B} = \begin{pmatrix} \Omega \cos \omega_d t \\ \Omega \sin \omega_d t \\ B_0 \end{pmatrix}$$

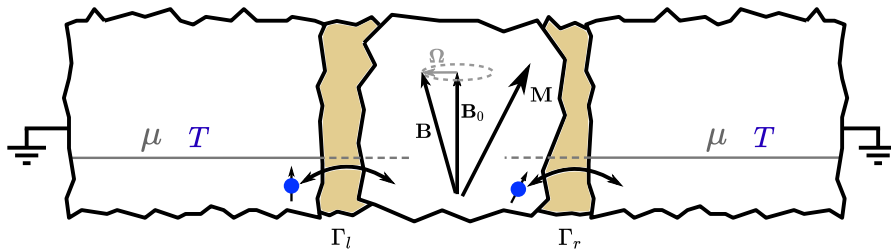
**FMR-driven
steady state
precessions**



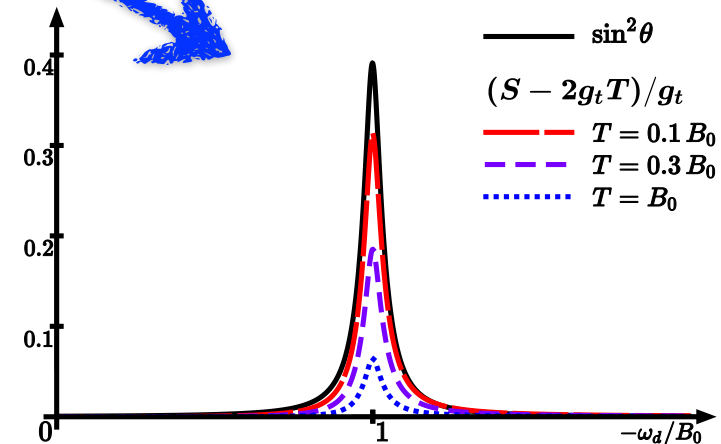
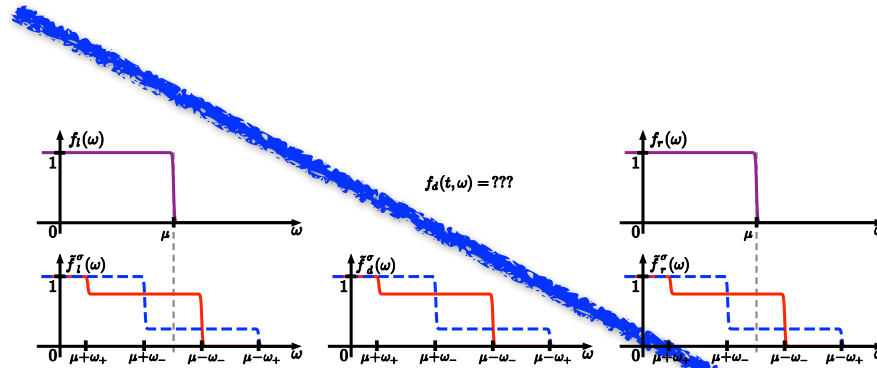
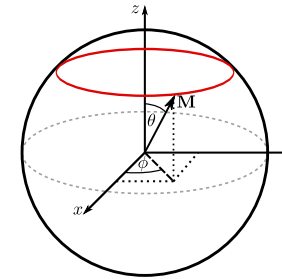
FMR driven micromagnet with a magnetic lead



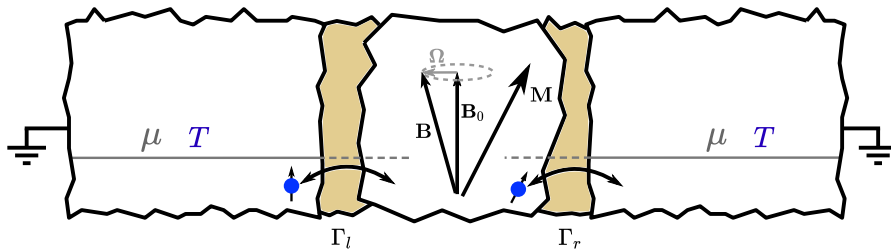
Summary



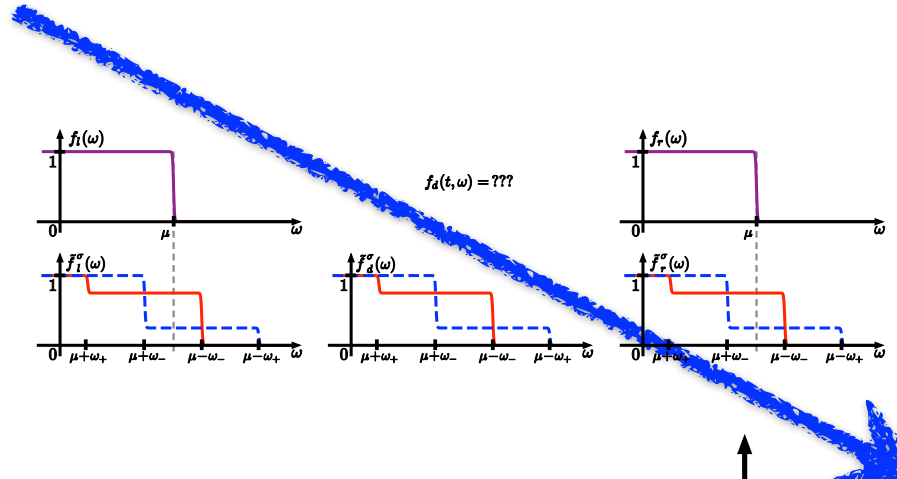
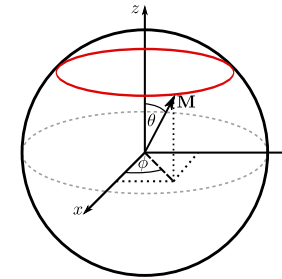
FMR-driven
steady state
precessions



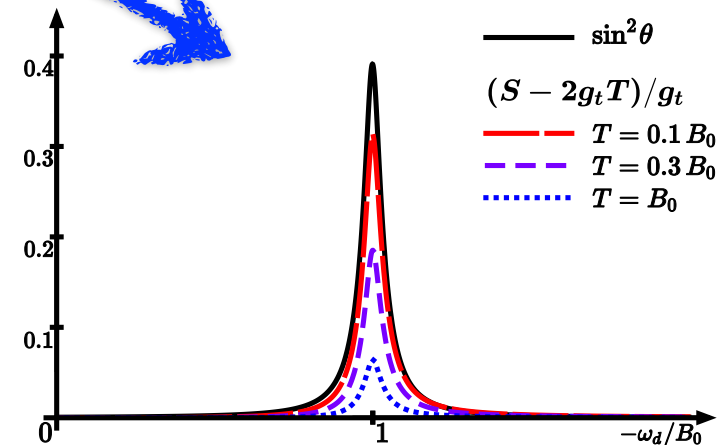
Summary



FMR-driven
steady state
precessions



Thank you
for your attention!



Equation of Motion - LLG+Slonczewski

$$\frac{d\mathbf{M}}{dt} = -\mathbf{B} \times \mathbf{M} - \alpha(\theta) \frac{\mathbf{M}}{M_0} \times \frac{d\mathbf{M}}{dt} + \mathbf{M} \times \left(\frac{\mathbf{I}_s}{S} \times \frac{\mathbf{M}}{M_0} \right)$$

Gilbert-damping $\alpha(\theta) = \frac{\tilde{g}(\theta)}{S}$

Spin-torque current $I_s = g_s V$

Charge current $I = 4g(\theta)V - g_s \sin^2 \theta \dot{\phi}$

Conductances

$$\tilde{g}(\theta) = \frac{\sin^2(\frac{\theta}{2})}{4} (g_{\uparrow\uparrow} + g_{\downarrow\downarrow}) + \frac{\cos^2(\frac{\theta}{2})}{4} (g_{\uparrow\downarrow} + g_{\downarrow\uparrow})$$

$$g_s = \frac{1}{4} (g_{\uparrow\uparrow} - g_{\downarrow\downarrow} - g_{\uparrow\downarrow} + g_{\downarrow\uparrow}) \quad g_{\sigma\sigma'} = 2\pi |t_l|^2 \rho_{dot}^\sigma \rho_{lead}^{\sigma'}$$

$$g(\theta) = \frac{\cos^2(\frac{\theta}{2})}{4} (g_{\uparrow\uparrow} + g_{\downarrow\downarrow}) + \frac{\sin^2(\frac{\theta}{2})}{4} (g_{\uparrow\downarrow} + g_{\downarrow\uparrow})$$

Gauge fixing

$$Q_{\parallel} = 0 \quad \longrightarrow \quad \dot{\chi} = \dot{\phi}(1 - \cos \theta)$$

Would be nice, but impossible
Berry phase different on two contours

$$\dot{\chi}_c(t) = \dot{\phi}_c(t) (1 - \cos \theta_c(t)) \quad \longrightarrow \quad Q_{\parallel,c} = 0$$

$$\chi_q(t) = \phi_q(t) (1 - \cos \theta_c(t))$$
$$Q_{\parallel,q} = \frac{1}{2} \sigma_z \sin \theta_c \left[\dot{\phi}_c \theta_q - \dot{\theta}_c \phi_q \right]$$

$$iS_{WZ} = iS \int dt \sin \theta_c \left[\dot{\phi}_c \theta_q - \dot{\theta}_c \phi_q \right] \quad \text{Keldysh Berry phase action}$$