Karlsruhe Institute of Technology

Vestigial order due to superconducting fluctuations in doped topological insulators

Jörg Schmalian
Karlsruhe Institute of Technology

collaborators

- **Matthias Hecker**
  Karlsruhe Institute of Technology

- **Rolf Lortz**
  Hong Kong University of S&T

Лев Петрович Горьков

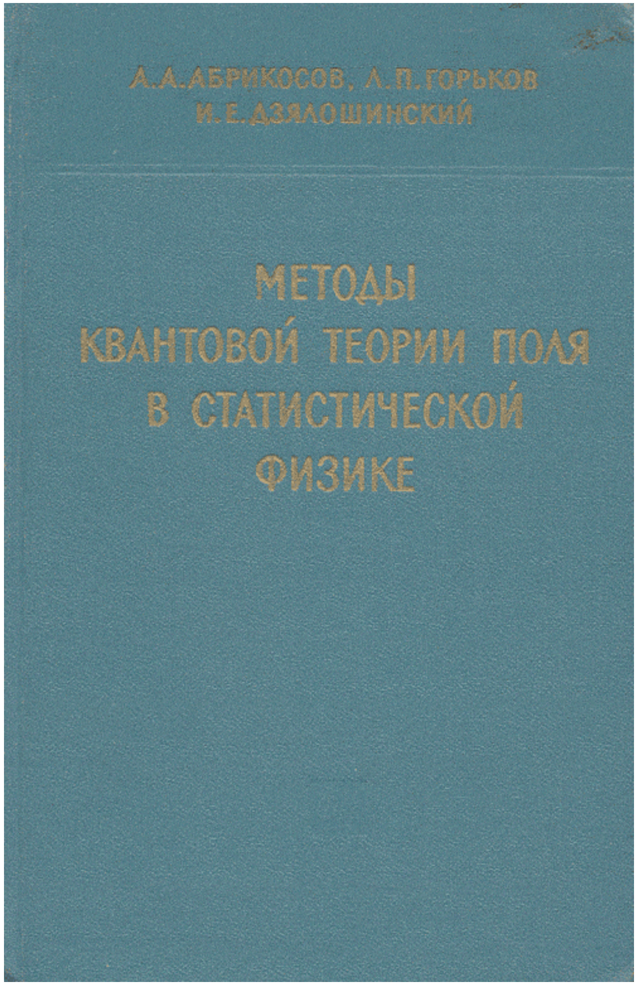
Tallahassee 2005

with David Pines in Urbana ~ 1991

German National Library in Leipzig

www.wikipedia.org

German National Library in Leipzig



a book that guided generations
all over the world

www.wikipedia.org

Symmetry classification of superconductors

Karlsruhe Institute of Technology

Anisotropy of the upper critical field in exotic superconductors

L. P. Gor'kov
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

(Submitted 17 September 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **40**, No. 8, 351–353 (25 October 1984)

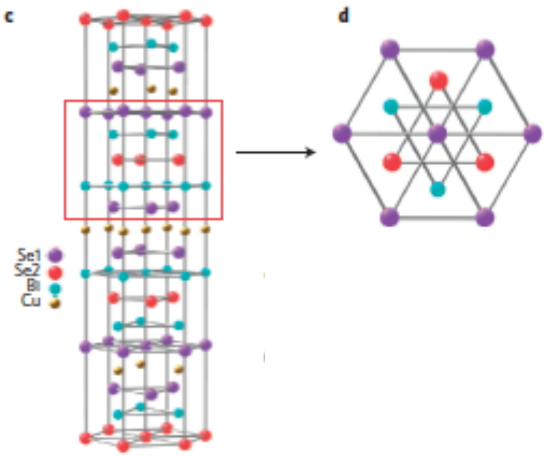
While the superconducting order parameter is characterized by a multidimensional representation of the rotation group (a state of the type ^{3}He-A), the upper critical field near T_c is anisotropic even in a crystal with a high degree of symmetry. This circumstance can be used to describe the nature of the observed superconductivity. The degeneracy can in principle be lifted by elastic stress.

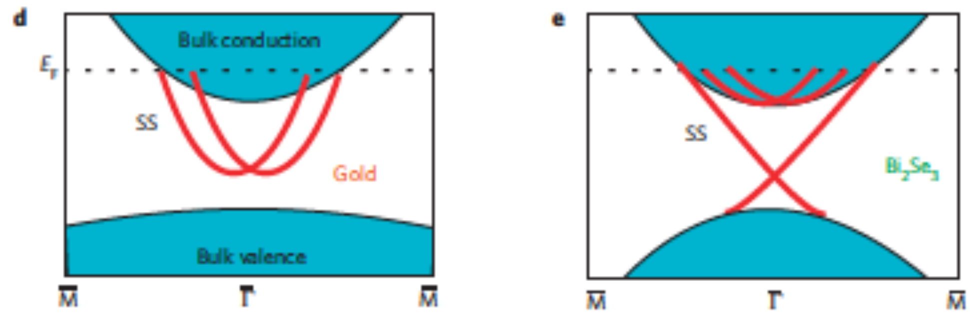
anisotropy of the upper critical field constrains the allowed symmetries of an unconventional superconductor

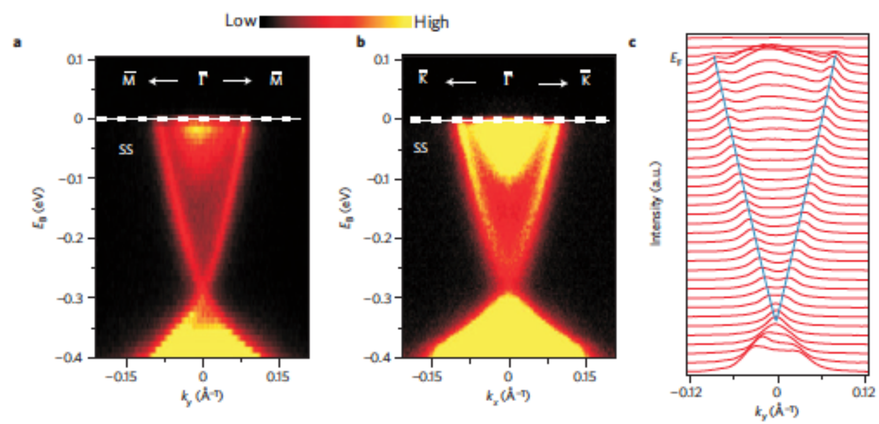
→ G. E. Volovik and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **88**, 1412 (1985)

Karlsruhe Institute of Technology

3d topological insulator Bi_2Se_3





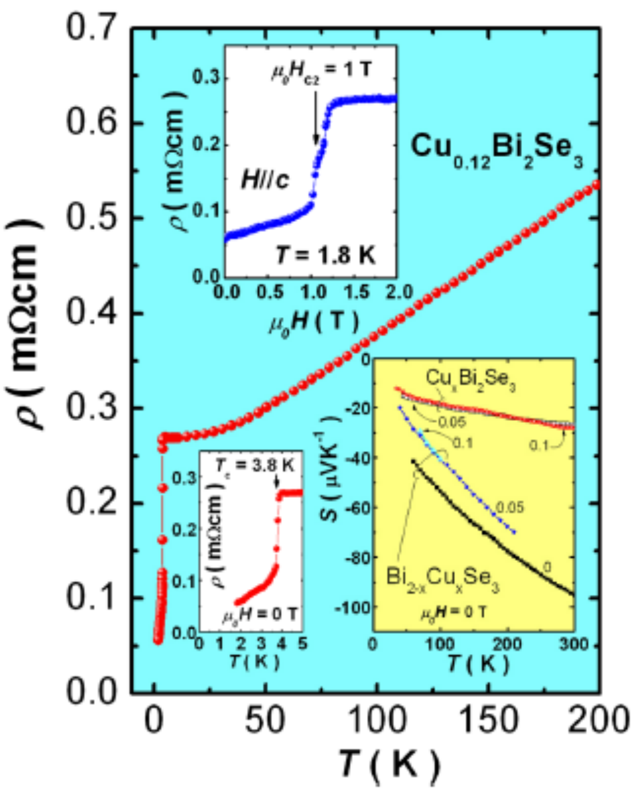


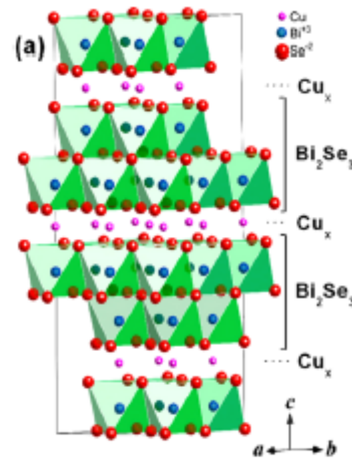
Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, A. Bansil, D. Grauer, Y.S. Hor, R.J. Cava, and M.Z. Hasan, Nature Phys. **5**, 398 (2009).

doping a 3d topological insulator Bi_2Se_3

superconductivity in $Cu_xBi_2Se_3$

Y. S. Hor, et al. Phys. Rev. Lett. **104,** 057001 (2010).
M. Kriener, et al., Phys. Rev. Lett. **106**, 127004 (2011).





$$T_c \sim 3\text{K}$$

$$n \sim 10^{20}\text{cm}^{-3}$$

low carrier concentration

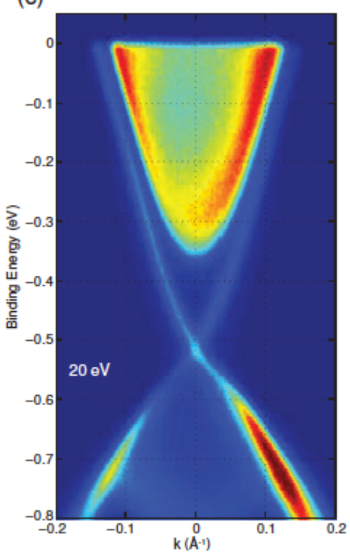
$$\xi_{ab}/\lambda_F \approx 2-4$$

short coherence length

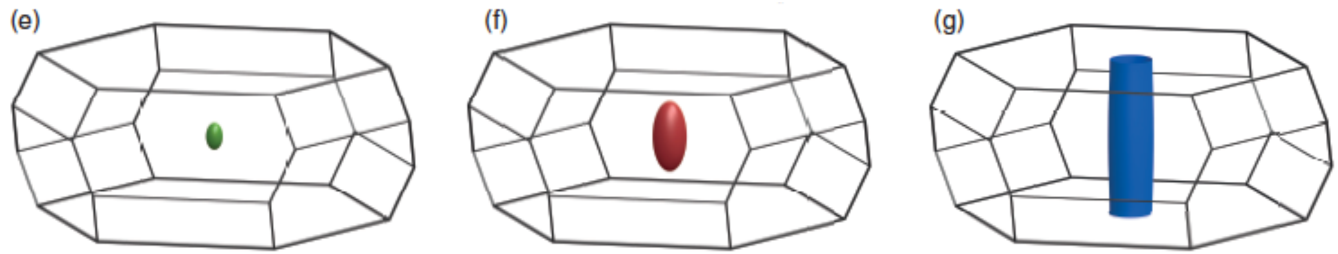
s.c. fluctuations

evolution of the Fermi surface with carrier concentration

ARPES



SdH



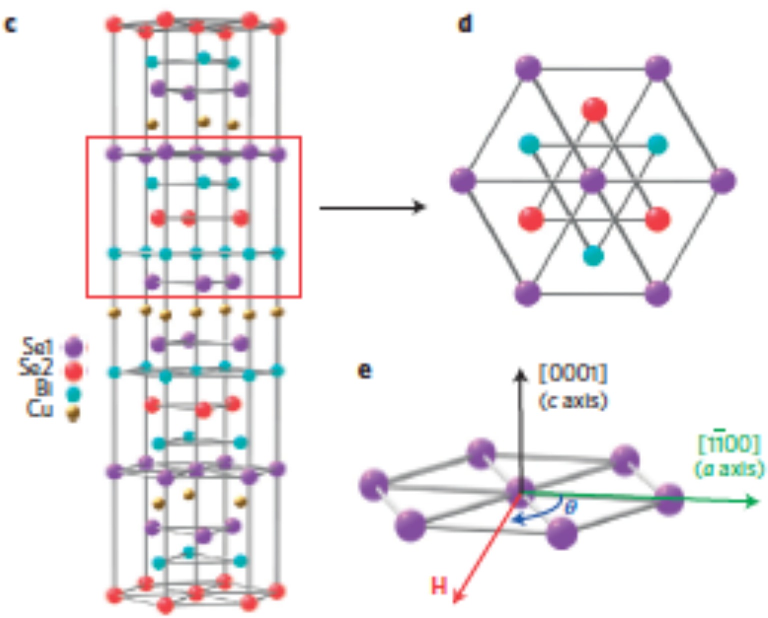
$n \sim 10^{17}\mathrm{cm}^{-3}$ $n \sim 10^{19}\mathrm{cm}^{-3}$ $n \sim 10^{20}\mathrm{cm}^{-3}$

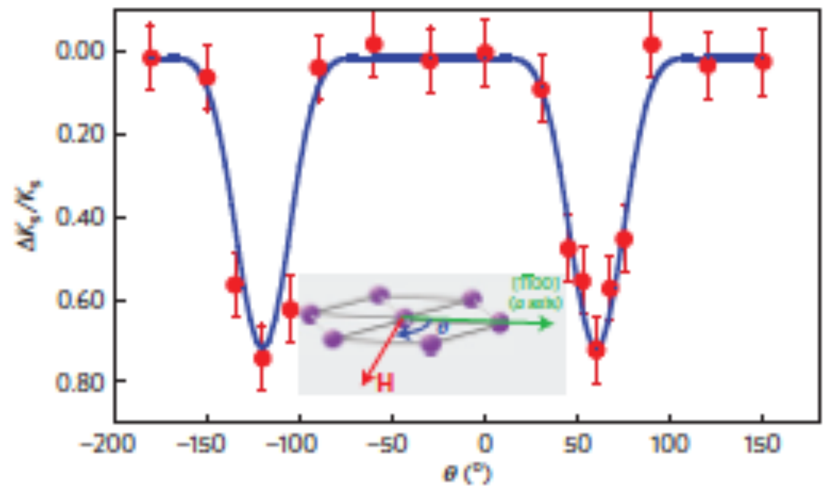
E. Lahoud, E. Maniv, M. Shaviv Petrushevsky, M. Naamneh, A. Ribak, S. Wiedmann, L. Petaccia, Z. Salman, K. B. Chashka, Y. Dagan, and A. Kanigel, Phys. Rev. B **88**, 195107 (2013)

electronic structure becomes increasingly anisotropic

rotational symmetry breaking

K. Matano, M. Kriener, K. Segawa, Y. Ando, and Guo-qing Zheng, Nature Physics **12**, 852 (2016).





nematic superconductor

classification of superconducting states

Landau (1937): order parameter transforms according to the irreducible representations of the symmetry group

point group D_{3d}

nematicity:
either E_g or E_u pairing

Character table for D_{3d} point group

	E	$2C_3$	$3C'_2$	i	$2S_6$	$3\sigma_d$	linear, rotations	quadratic
A_{1g}	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	-1	1	1	-1	R_z	
E_g	2	-1	0	2	-1	0	(R_x, R_y)	(x^2-y^2, xy) (xz, yz)
A_{1u}	1	1	1	-1	-1	-1		
A_{2u}	1	1	-1	-1	-1	1	z	
E_u	2	-1	0	-2	1	0	(x, y)	

L. Fu and E. Berg, Phys. Rev. Lett. **105**,4 097001 (2010).

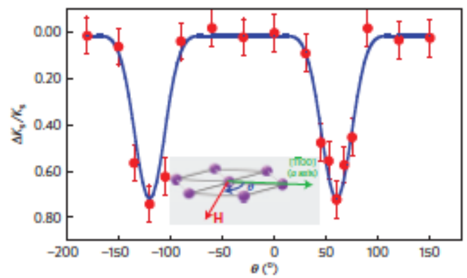
J. W. F. Venderbos, V. Kozii, and L. Fu, B **94**, 180504(R) (2016)

odd-parity topological superconductor?

Two-component superconducting states

E_g, singlet: $\Delta_{\mathbf{p}} = \Delta_x \left(p_x p_y + \eta p_y p_z\right) + \Delta_y \left(p_x^2 - p_y^2 + \eta p_x p_z\right)$

E_u, triplet: $\mathbf{d}_{\mathbf{p}} = \Delta_x \left(\hat{\mathbf{x}} p_z - \eta \hat{\mathbf{z}} p_x\right) + \Delta_y \left(\hat{\mathbf{y}} p_z - \eta \hat{\mathbf{z}} p_y\right)$



two options:

chiral

$$(\Delta_x, \Delta_y) = \frac{\Delta_0}{\sqrt{2}}(1, \pm i)$$

chiral superconductor
(breaks time-reversal symmetry)

nematic

$$(\Delta_x, \Delta_y) = \Delta_0(\cos\theta_n, \sin\theta_n)$$

$\theta_n = \frac{\pi}{2} + n\frac{\pi}{3}$ or $\theta_n = n\frac{\pi}{3}$

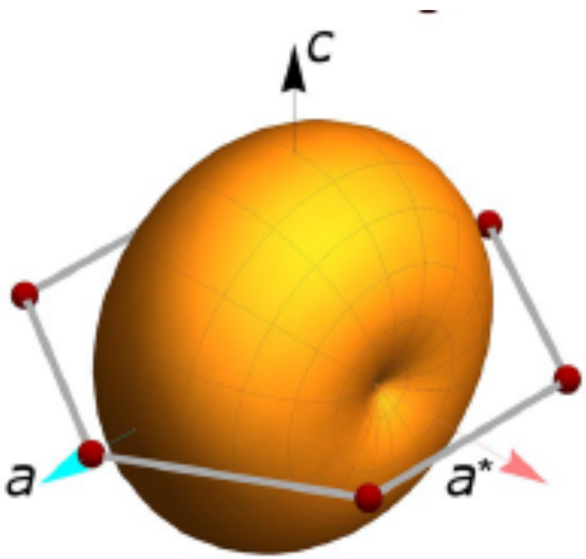
nematic superconductor
(breaks rotation symmetry)

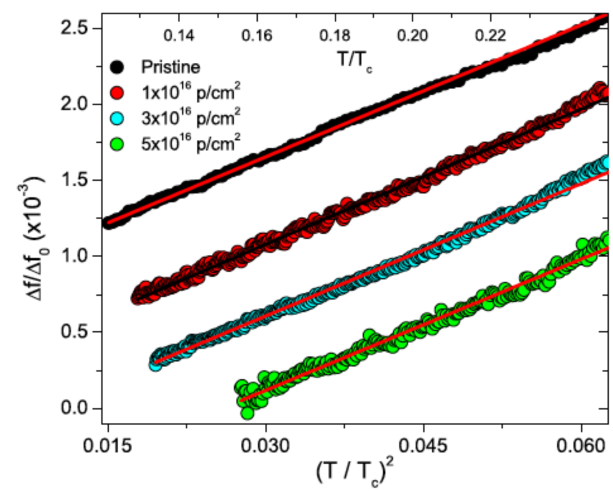
Two-component superconducting states

singlet: $\Delta_{\mathbf{p}} = \Delta_x \left(p_x p_y + \eta p_y p_z \right) + \Delta_y \left(p_x^2 - p_y^2 + \eta p_x p_z \right)$

E_u, triplet: $\mathbf{d}_{\mathbf{p}} = \Delta_x \left(\hat{\mathbf{x}} p_z - \eta \hat{\mathbf{z}} p_x \right) + \Delta_y \left(\hat{\mathbf{y}} p_z - \eta \hat{\mathbf{z}} p_y \right)$

strong evidence for point nodes

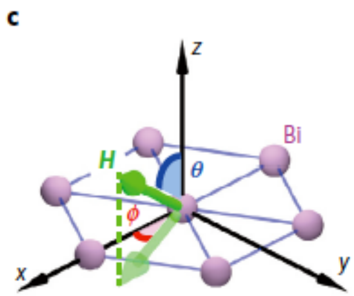




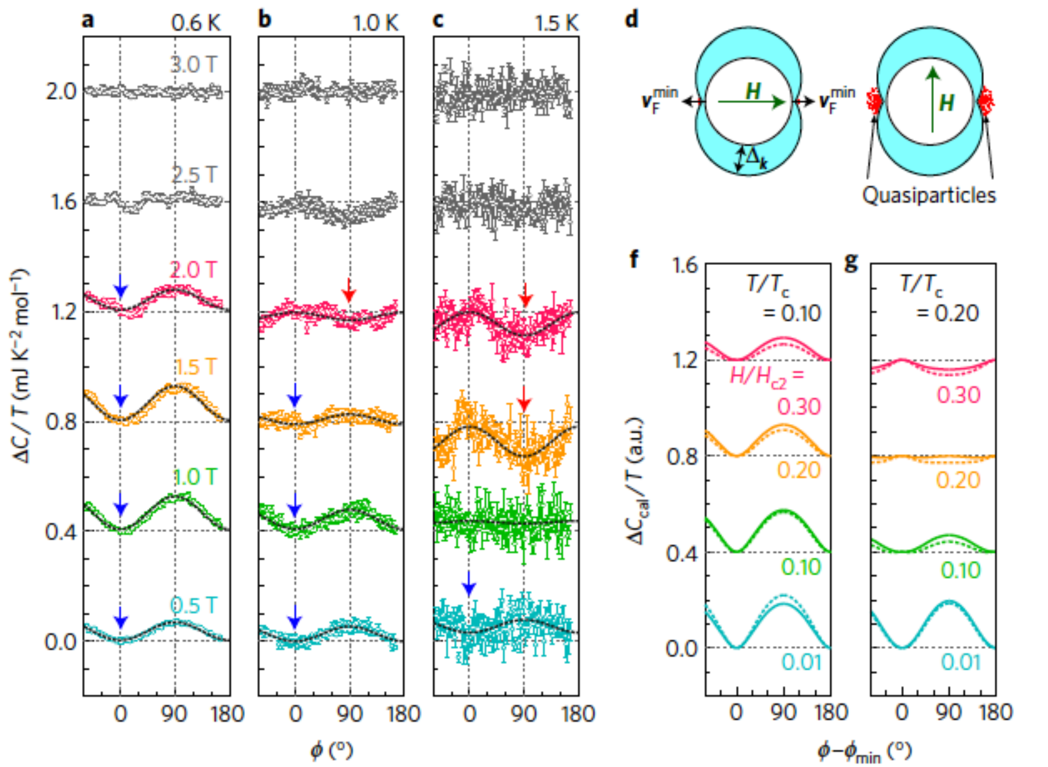
M. P. Smylie K. Willa, H. Claus, A. E. Koshelev , K. W. Song, W.-K. Kwok, Z. Islam, G. D. Gu, J. A. Schneeloch, R. D. Zhong & U. Welp, PRB **96**, 115145 (2017); ibid. Scientific Reports **8**, 7666| (2018)

anisotropic thermodynamic response

Shingo Yonezawa, Kengo Tajiri, Suguru Nakata, Yuki Nagai, ZhiweiWang, Kouji Segawa, Yoichi Ando, and Yoshiteru Maeno Nat. Phys. **13**, 123 (2017).



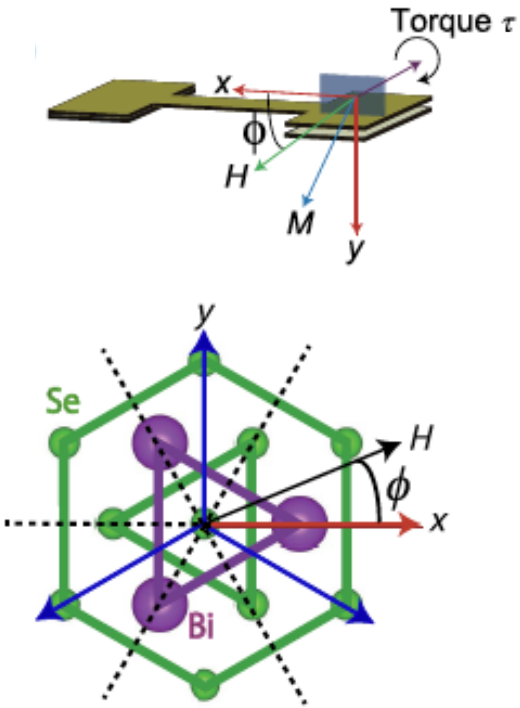
heat capacity as function of in-plane magnetic field

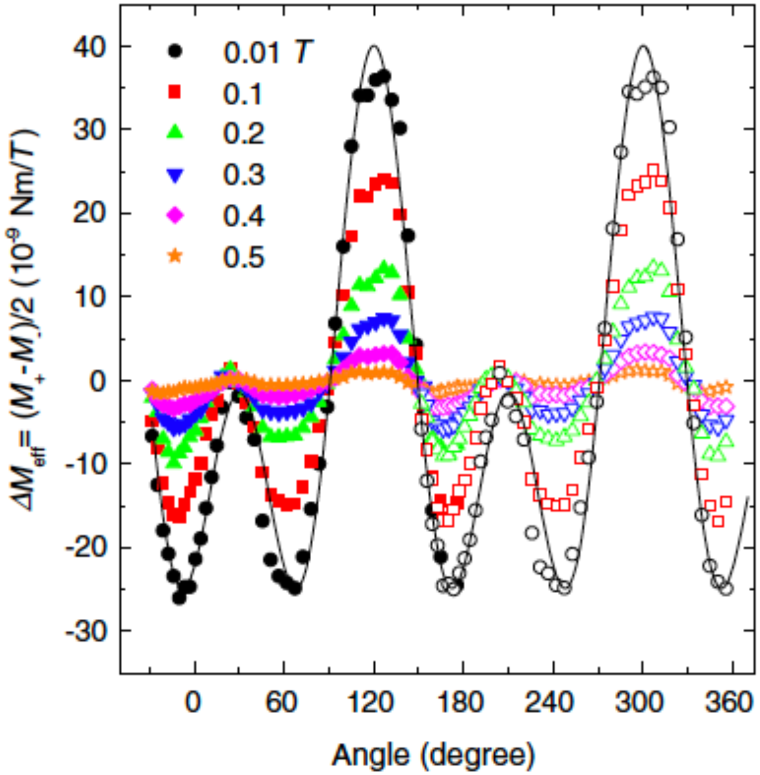


nematic superconductor

anisotropic torque

T. Asaba, J. Lawson, C. Tinsman, L. Chen, P. Corbae, G. Li, Y. Qiu,
Y. S. Hor, L. Fu, and L. Li, Phys. Rev. X **7**, 011009 (2017)



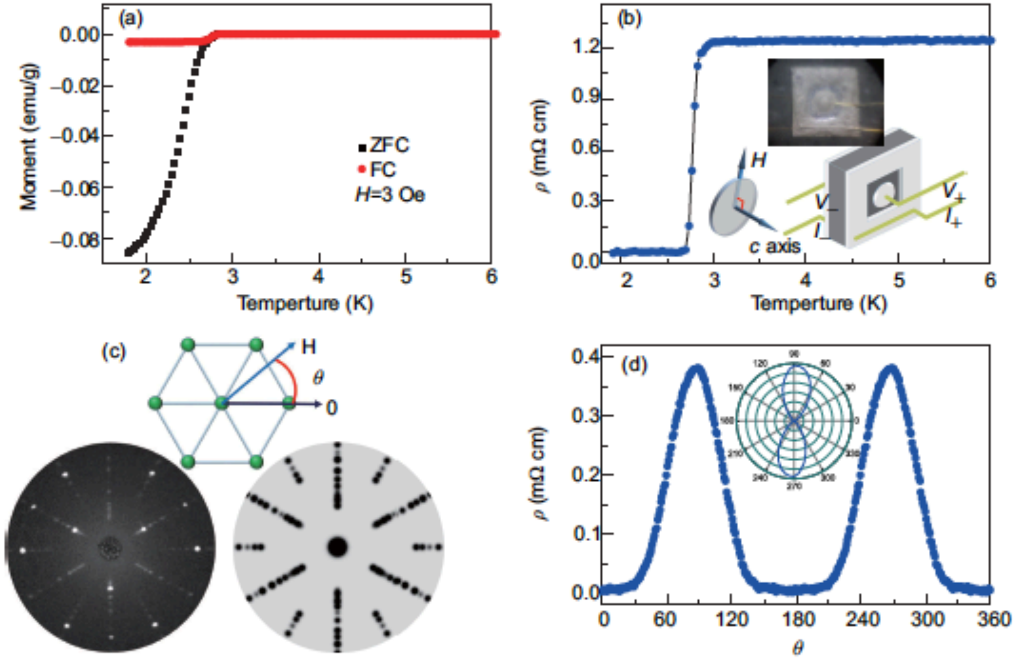


six and two-fold symmetric susceptibility

anisotropic c-axis transport

Guan Du, YuFeng Li, J. Schneeloch, R. D. Zhong, GenDa Gu, Huan Yang, Hai Lin, and Hai-Hu Wen
Sci. China Phys. Mech. Astron. **60**, 037411 (2017).

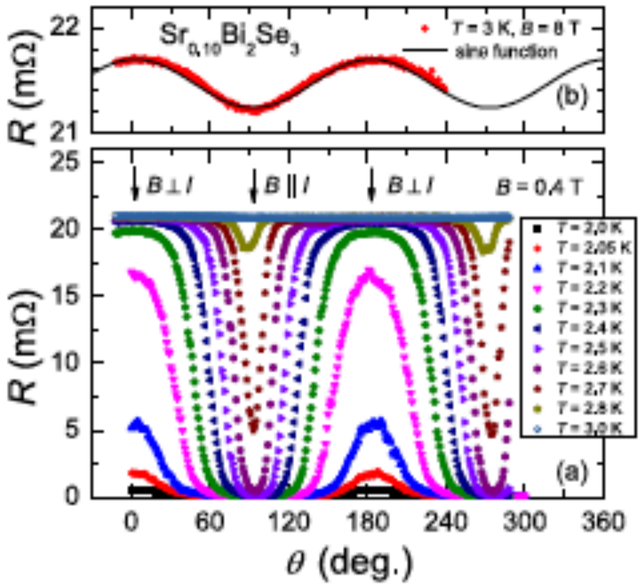
anisotropic c-axis resistivity as function of in-plane magnetic field

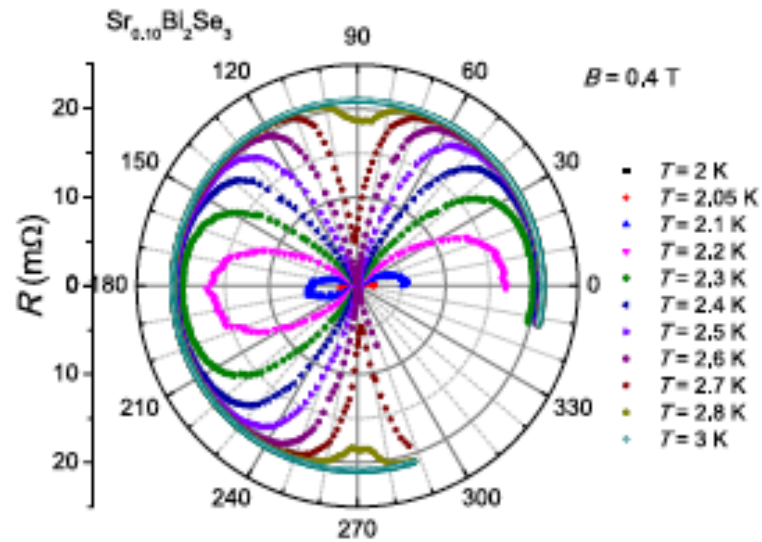


nematic superconductor

anisotropic upper critical field

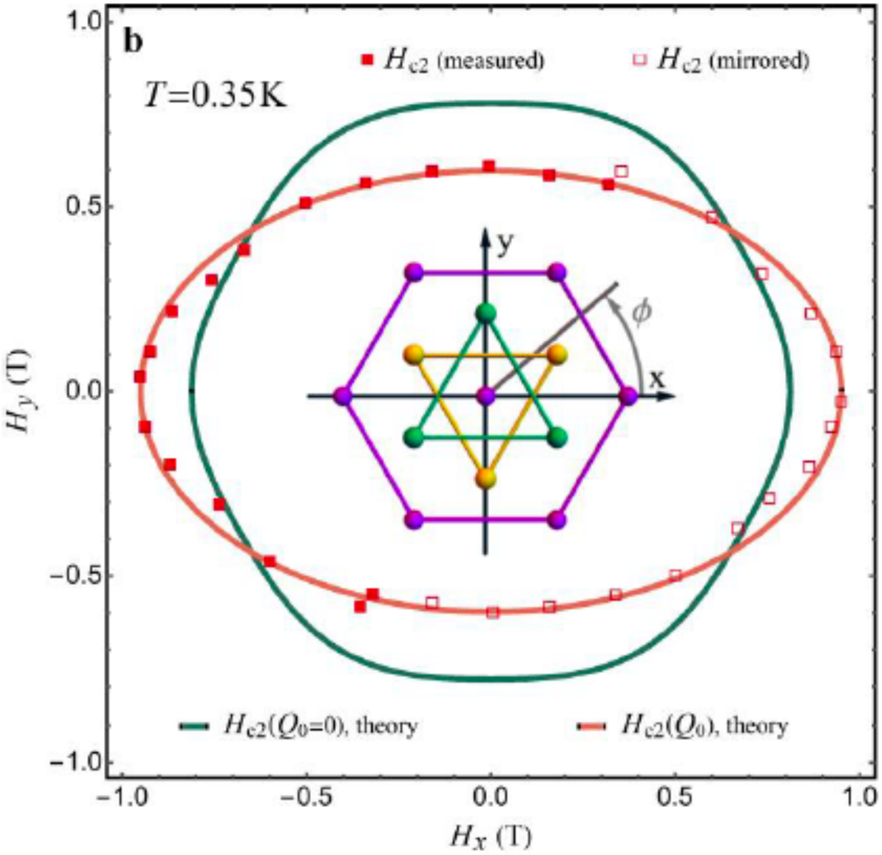
Y. Pan , A. M. Nikitin , G. K. Araizi , Y. K. Huang , Y. Matsushita , T. Naka & A. de Visser.
Sci. Rep. 6, 28632 (2016).





Problem: upper-critical field should have six-fold symmetry
P. L. Krotkov and V. P. Mineev, Phys. Rev. B **65**, 224506 (2002)

anisotropic upper critical field



Z_3-symmetry already broken at the transition

→two-fold symmetric upper-critical field

include superconducting fluctuations

Karlsruhe Institute of Technology

$$S_\Delta = S_\Delta^{\text{grad}} + \int_r \left(r_0 \Delta^\dagger \Delta + u \left(\Delta^\dagger \Delta\right)^2 + v \left(\Delta^\dagger \tau_y \Delta\right)^2 \right)$$

form bilinear combinations
that transform non-trivially

$$\hat{Q} = \begin{pmatrix} q_1 & q_2 \\ q_2 & -q_1 \end{pmatrix}$$

collective variable that can
independently condense

$$q_1 = \Delta_x^* \Delta_x - \Delta_y^* \Delta_y$$

$$q_2 = \Delta_x^* \Delta_y + \Delta_y^* \Delta_x$$

collective nematic fluctuations

$$S = \frac{1}{8}\int_x \left(\frac{1}{v}\mathrm{tr}\left(\hat{Q}\hat{Q}\right) - \frac{1}{u+v}\mathrm{tr}\left(\hat{Q}_0\hat{Q}_0\right)\right) + \int_k \mathrm{tr}\log\chi_k[\hat{Q}]^{-1}.$$

s.c. fluctuations

nematic order parameter $\langle Q_{\alpha\beta}\rangle = Q\left(n_\alpha n_\beta - \frac{1}{2}\delta_{\alpha\beta}\right)$

$\propto \left\langle |\Delta|^2 \right\rangle$ due to s.c. fluctuations

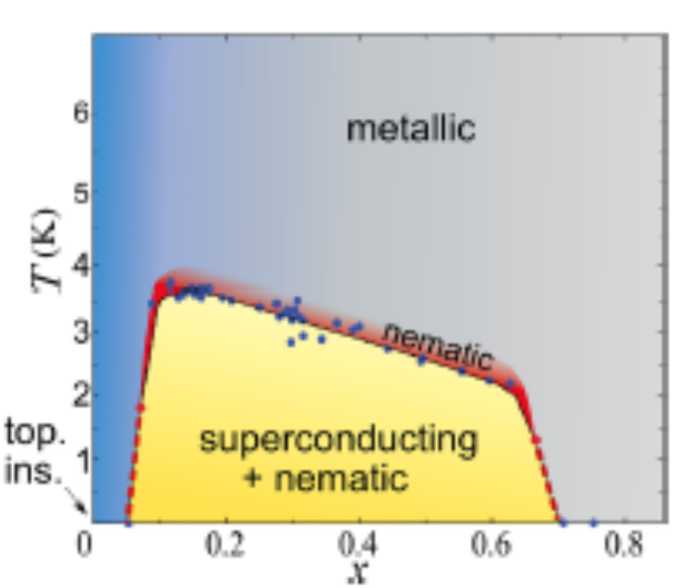
$\mathbf{n} = (\cos\theta_n, \sin\theta_n)$ nematic director

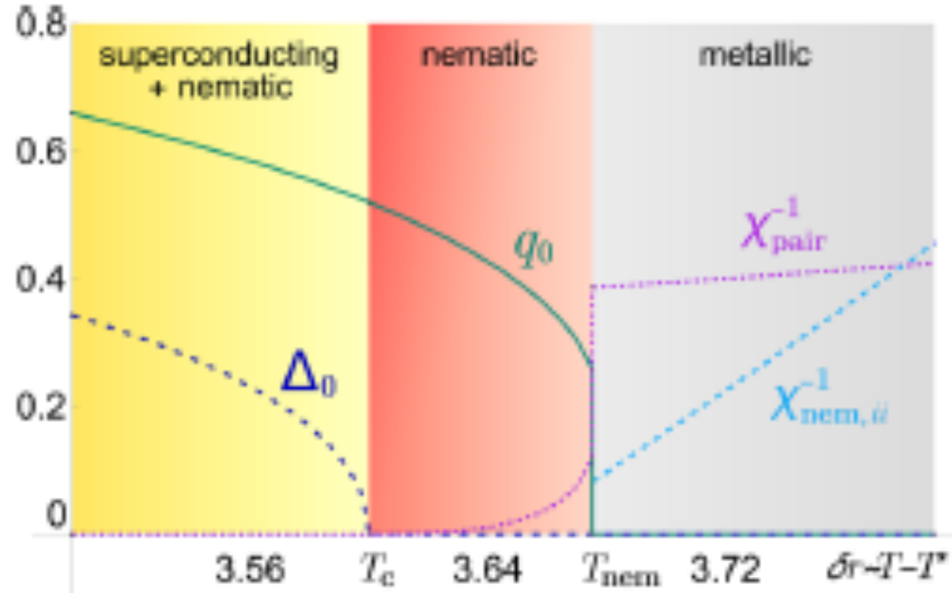
small-Q expansion $\hat{Q} = \begin{pmatrix} q_1 & q_2 \\ q_2 & -q_1 \end{pmatrix}$

$$S = S_{\mathrm{grad}} + \int_x \left(\frac{r}{2}\left(q_1^2+q_2^2\right) - \frac{g}{3}q_1\left(q_1^2 - 3q_2^2\right) + \frac{g'}{4}\left(q_1^2+q_2^2\right)^2\right)$$

three state Potts model J. P. Straley & M. E. Fisher, J. Phys. A **6**, 1310 (1973).

Nematic order above T_c





nematic transition in the 3 states Potts model class (1st order)

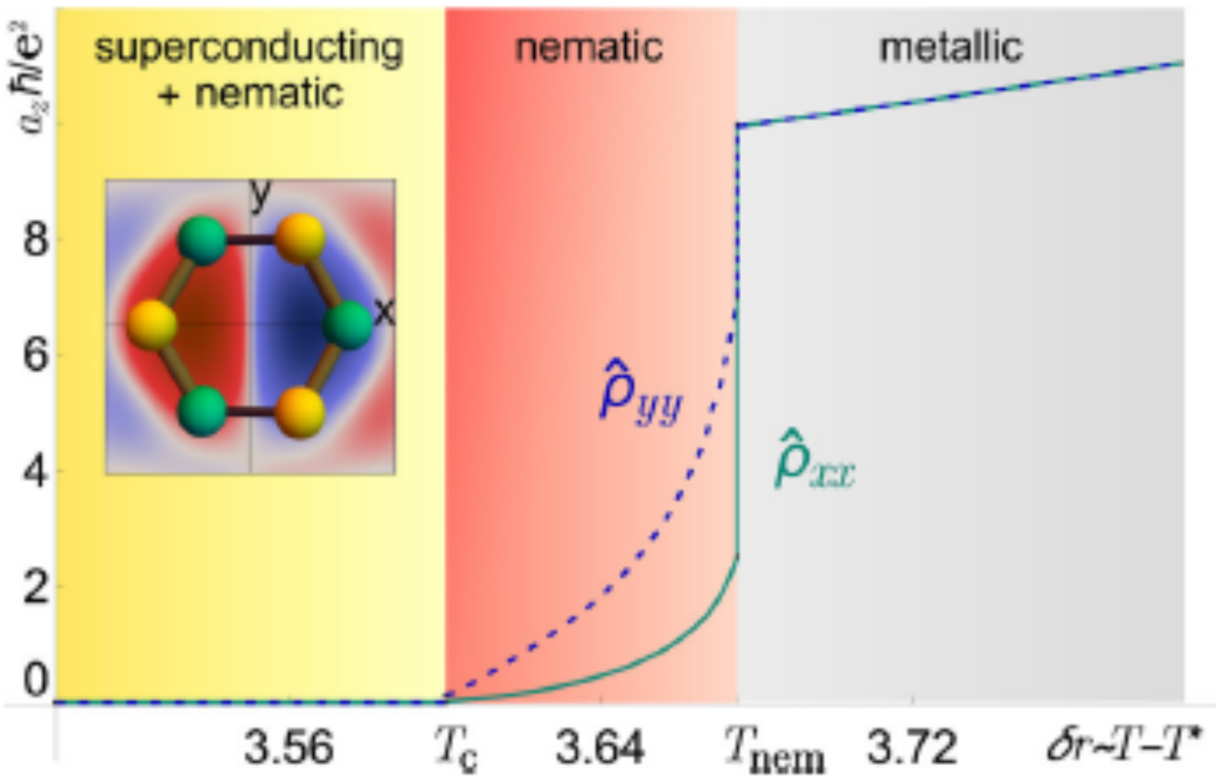
- dramatic increase of the pair susceptibility at the nematic transition
- softening of nematic fluctuations above T_{nem}

Matthias Hecker and J.S., npj Quantum Mater. **3**, 26 (2018).

implications: i) transport anisotropy

Broken rotation symmetry+ large superconducting fluctuations

→ **large anisotropic para-conductivity**



Matthias Hecker and J.S., npj Quantum Mater. **3**, 26 (2018).

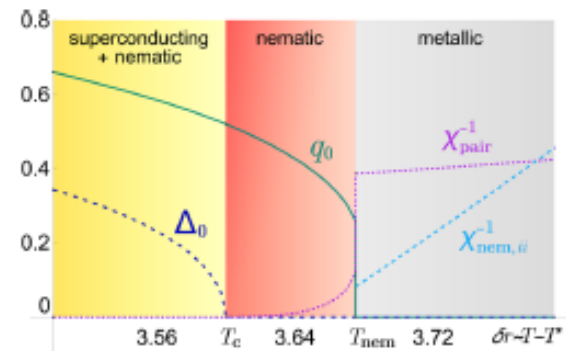
implications: ii) lattice softening + Raman

coupling to strain

$$H \to H + \lambda \int d^3x \sum_{\alpha,\beta} Q_{\alpha,\beta}\epsilon_{\beta,\alpha} + \frac{1}{4}\int_x C^0_{E_g}\left[\left(\varepsilon_{xx}-\varepsilon_{yy}\right)^2 + 4\varepsilon_{xy}\right].$$

softening of the elastic modulus

$$1/C_{E_g} = 1/C^{(0)}_{E_g} + \frac{\lambda^2}{2C^{(0)}_{E_g}}\mathrm{tr}\hat{\chi}_{\mathrm{nem}}$$

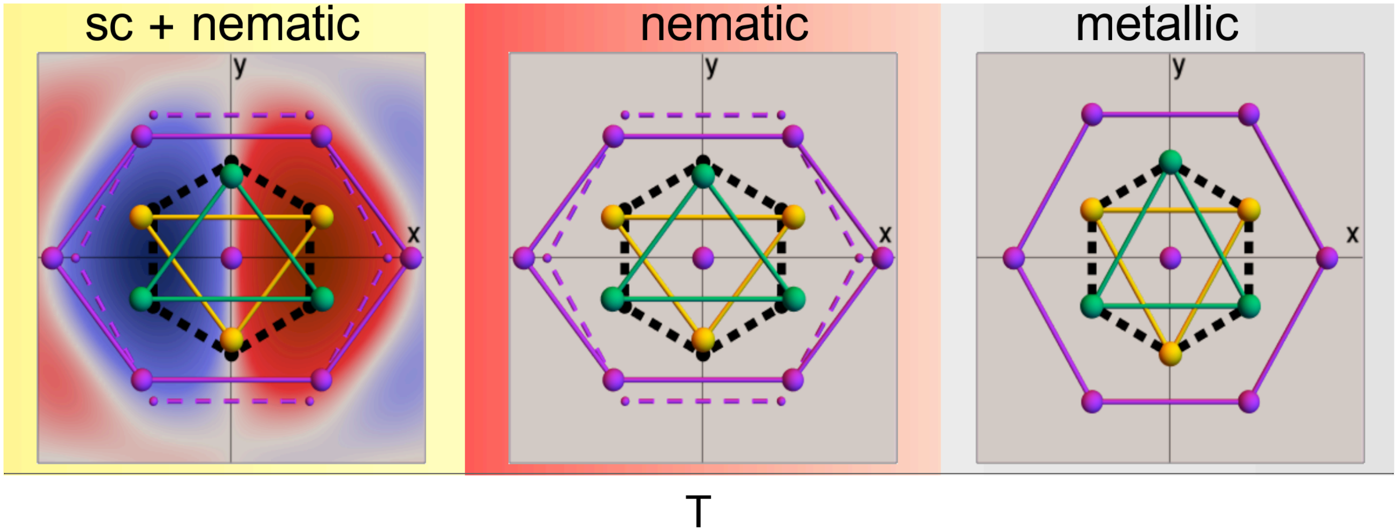


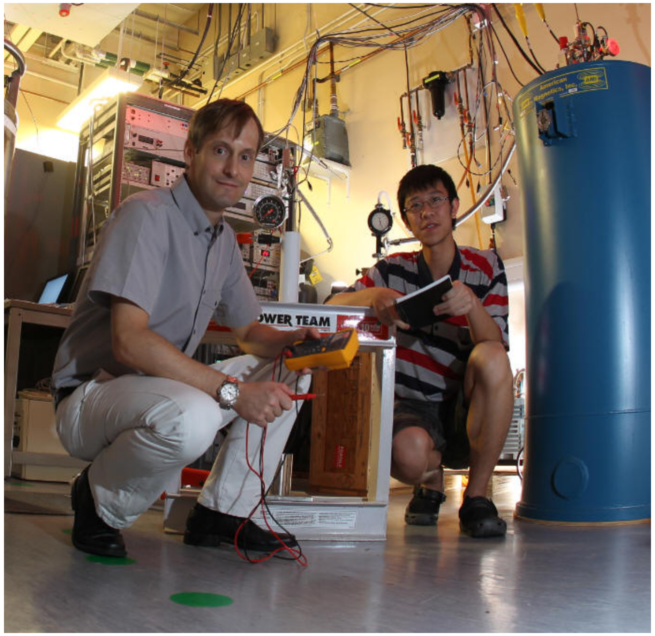
enhancement of the electronic Raman response in the E_g-channel

Matthias Hecker and J.S., npj Quantum Mater. **3**, 26 (2018).

implications: iii) lattice distortion

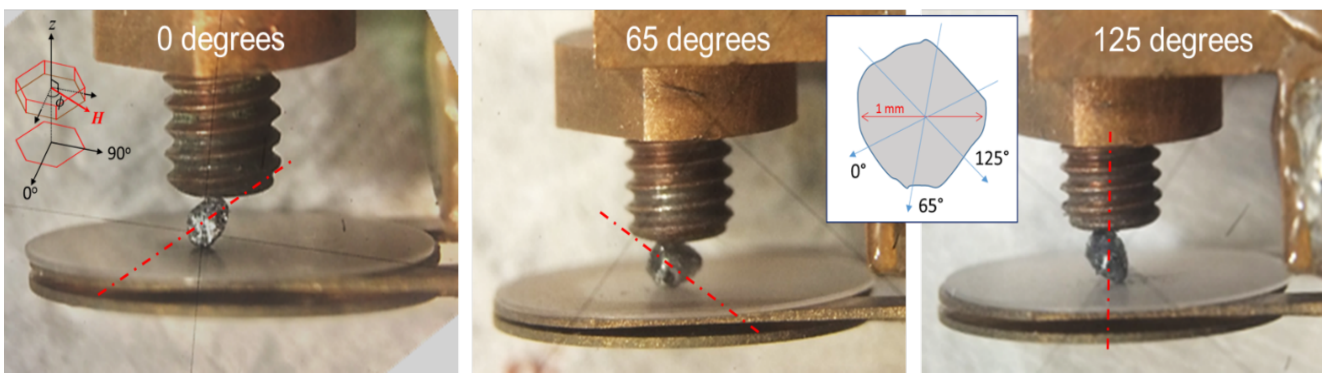
below T_{nem} : distortion of the lattice





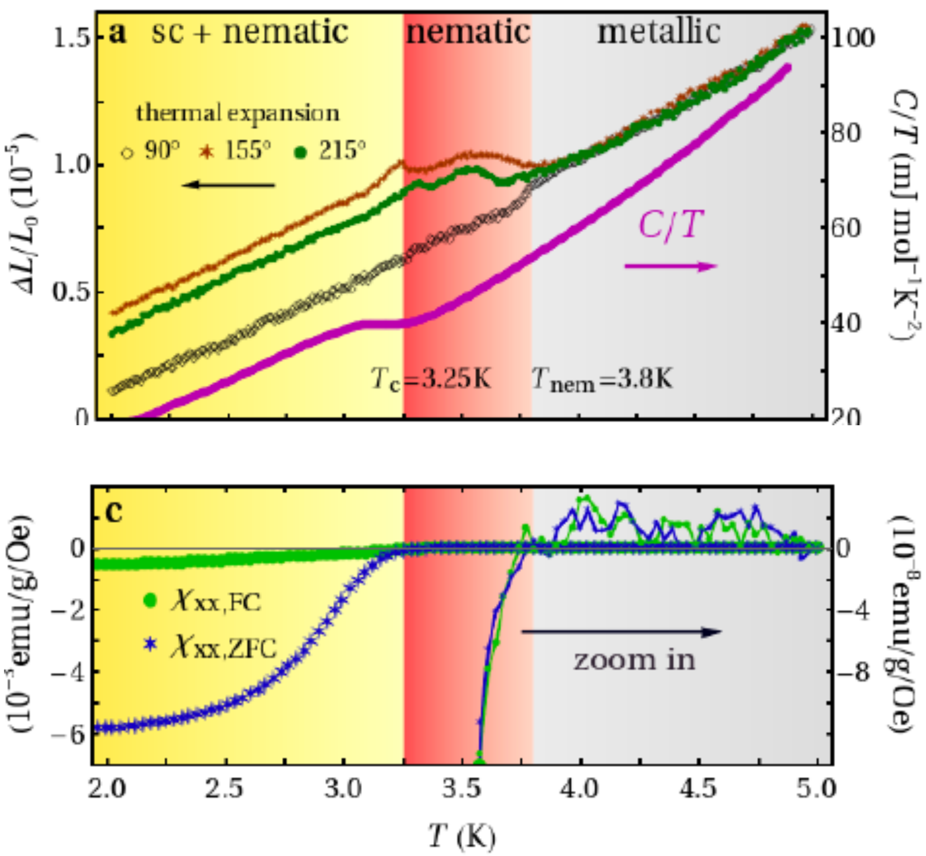
Rolf Lortz
The Hong Kong University
of Science & Technology

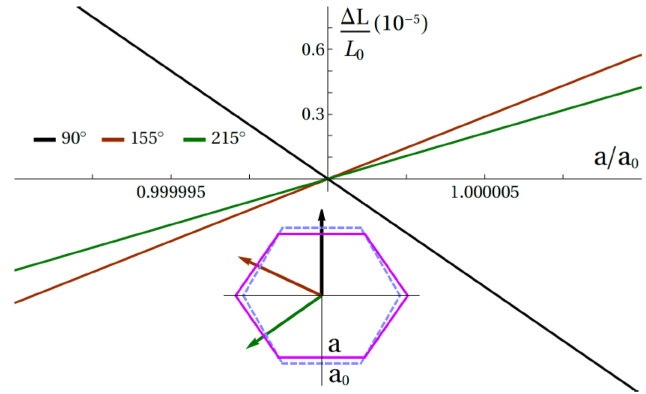
$Nb_{0.25}Bi_2Se_3$ single crystal mounted in the capacitive dilatometer along the three measured directions within the Bi_2Se_3 basal plane



Thermal expansion and magnetization measurements

Karlsruhe Institute of Technology





strong evidence for vestigial superconducting phase !

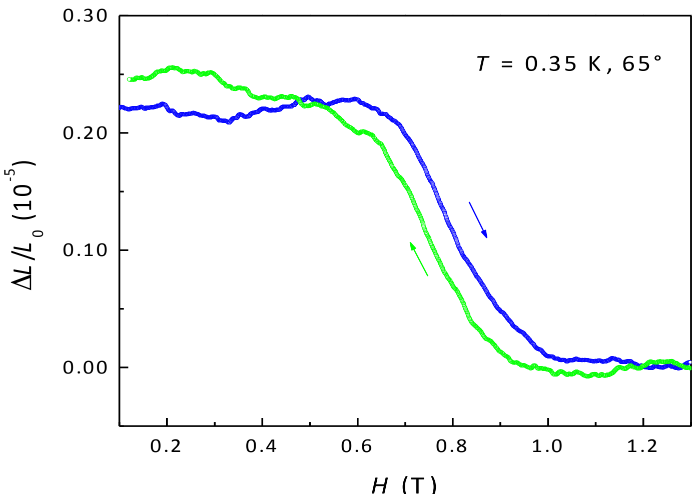
$$\langle \Delta_{x,y} \rangle = 0$$

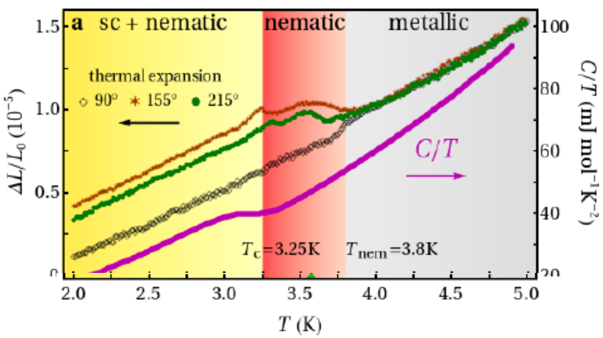
$$\langle \Delta_x^* \Delta_x - \Delta_y^* \Delta_y \rangle \neq 0$$

condensation of anisotropic s.c. fluctuations

C.-w. Cho, J. Shen, J. Lyu, S. H. Lee, Yew San Hor, M. Hecker, J. S., and R. Lortz, arXiv:1905.01702

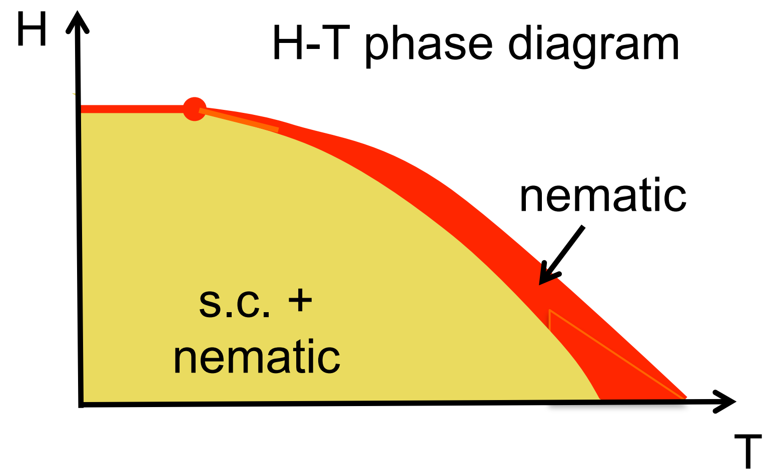
magneto-striction measurements

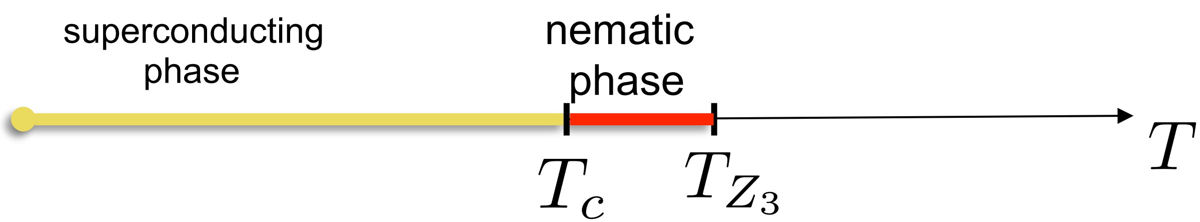




magnetic field ~ H_{c2} restores rotation invariance

nematic state ←→ s.c. state





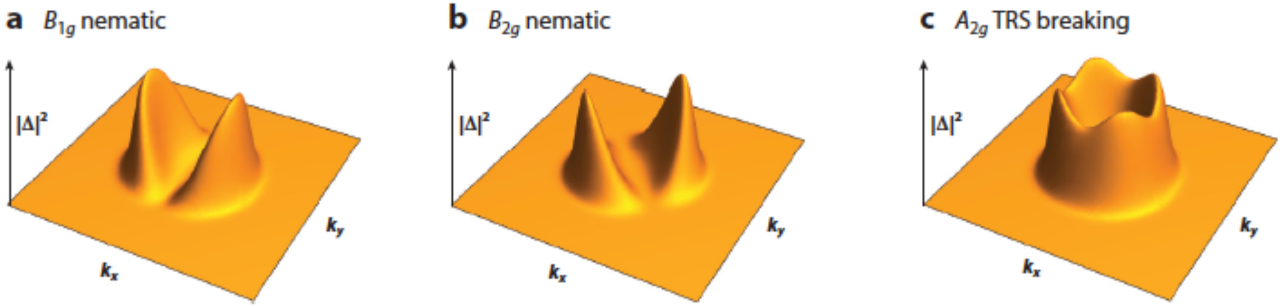
C.-w. Cho, J. Shen, J. Lyu, S. H. Lee, Yew San Hor, M. Hecker, J. S., and R. Lortz, arXiv:1905.01702

Vestigial phases in superconductors:
Example: p-wave superconductivity in a tetragonal crystal

superconducting order parameter:

$$\langle c_{\mathbf{p}\alpha} c_{-\mathbf{p}\beta} \rangle = \left[\mathbf{d}_{\mathbf{p}} \cdot \boldsymbol{\sigma} i \sigma^y \right]_{\alpha\beta} \qquad \mathbf{d}_{\mathbf{p}} = \hat{\mathbf{z}} \left(\Delta_x p_x + \Delta_y p_y \right)$$

(Δ_x, Δ_y)	$(1,0),(0,1)$	$(1,\pm 1)$	$(1,\pm i)$
	p_x, p_y	$p_x \pm p_y$	$p_x \pm i p_y$



Vestigial phases in superconductors:

Example: p-wave superconductivity in a tetragonal crystal

Karlsruhe Institute of Technology

superconducting order parameter:

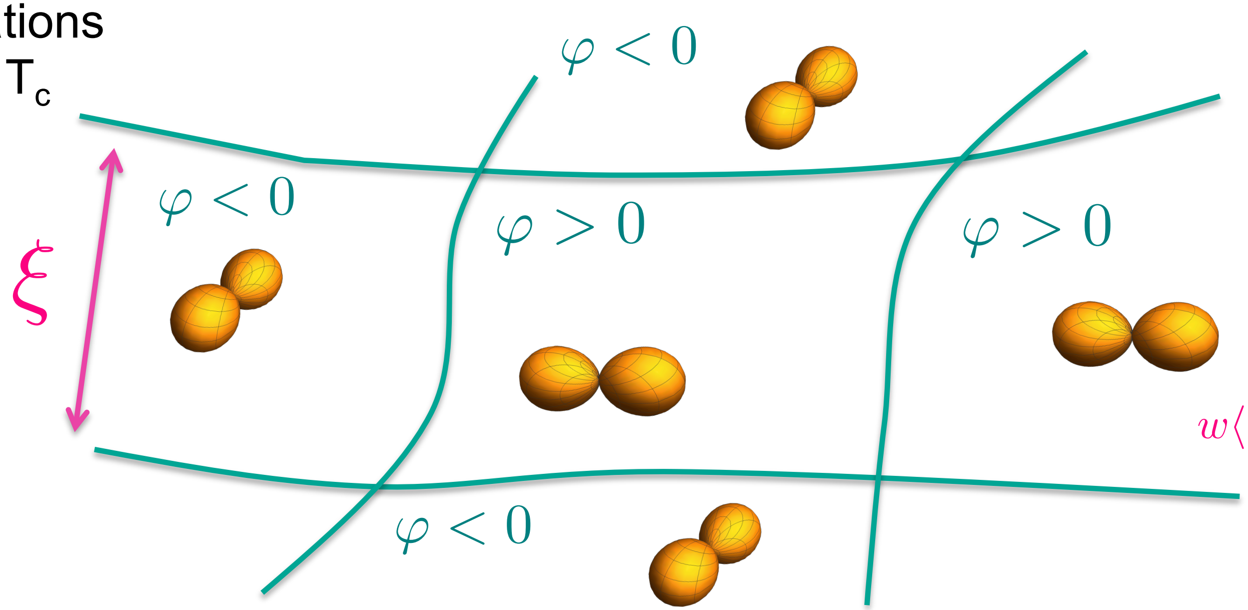
$$\langle c_{\mathbf{p}\alpha} c_{-\mathbf{p}\beta} \rangle = [\mathbf{d}_\mathbf{p} \cdot \boldsymbol{\sigma} i \sigma^y]_{\alpha\beta} \qquad \mathbf{d}_\mathbf{p} = \hat{\mathbf{z}} \left(\Delta_x p_x + \Delta_y p_y \right)$$

(Δ_x, Δ_y)

$(1,0), (0,1)$
p_x, p_y

$$E = -w \int d^d x \left(\Delta_x^2 - \Delta_y^2 \right)^2$$

fluctuations above T_c



Ising variable

$$\varphi = \langle \Delta_x^2 - \Delta_y^2 \rangle \neq 0$$

orders above T_c

$$w \langle \Delta^2 \rangle^2 (T_{\mathrm{Ising}})^2 \xi^d \sim T_{\mathrm{Ising}}$$

Vestigial phases in superconductors:
Example: p-wave superconductivity in a tetragonal crystal

Karlsruhe Institute of Technology

superconducting order parameter:

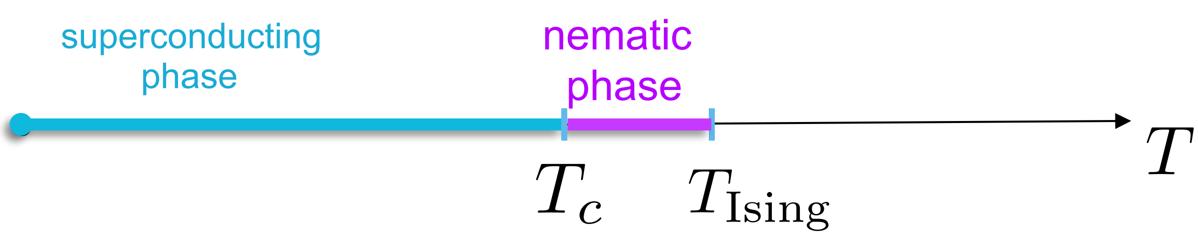
$$\langle c_{\mathbf{p}\alpha} c_{-\mathbf{p}\beta} \rangle = \left[\mathbf{d}_{\mathbf{p}} \cdot \boldsymbol{\sigma} i \sigma^{y} \right]_{\alpha\beta} \qquad \mathbf{d}_{\mathbf{p}} = \hat{\mathbf{z}} \left(\Delta_x p_x + \Delta_y p_y \right)$$

(Δ_x, Δ_y)

$(1,0), (0,1)$

p_x, p_y

$$E = -w \int d^d x \left(\Delta_x^2 - \Delta_y^2 \right)^2$$



Ising variable

$$\varphi = \langle \Delta_x^2 - \Delta_y^2 \rangle \neq 0$$

orders above T_c

$$w \langle \Delta^2 \rangle^2 (T_{\text{Ising}})^2 \xi^d \sim T_{\text{Ising}}$$

split transition robust in two-dimensional and strongly anisotropic systems

formal analysis: nonlinear σ model, renormalization group, self-consistent Gaussian, large-N expansions

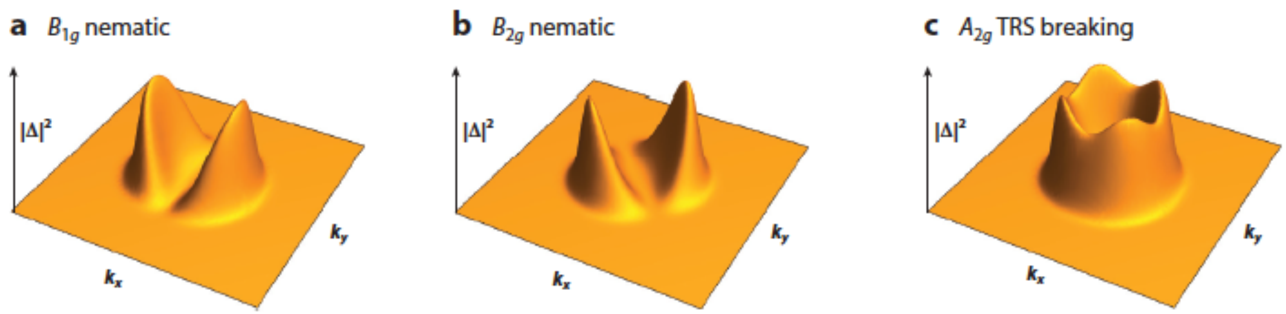
Vestigial phases in superconductors:

Example: p-wave superconductivity in a tetragonal crystal

superconducting order parameter:

$$\langle c_{\mathbf{p}\alpha} c_{-\mathbf{p}\beta} \rangle = \left[\mathbf{d_p} \cdot \boldsymbol{\sigma} i \sigma^y \right]_{\alpha\beta} \qquad \mathbf{d_p} = \hat{\mathbf{z}} \left(\Delta_x p_x + \Delta_y p_y \right)$$

(Δ_x, Δ_y) $\quad (1,0),(0,1)$ $\quad (1,\pm 1)$ $\quad (1,\pm i)$



x^2-y^2 **nematic** | xy **nematic** | **time reversal** symmetry

$$\varphi = \Delta_x^* \Delta_x - \Delta_y^* \Delta_y \qquad \varphi = \Delta_x^* \Delta_y + \Delta_y^* \Delta_x \qquad \varphi = i \left(\Delta_x^* \Delta_y - \Delta_y^* \Delta_x \right)$$

there is no ordinary second order transition to a multicomponent superconductor

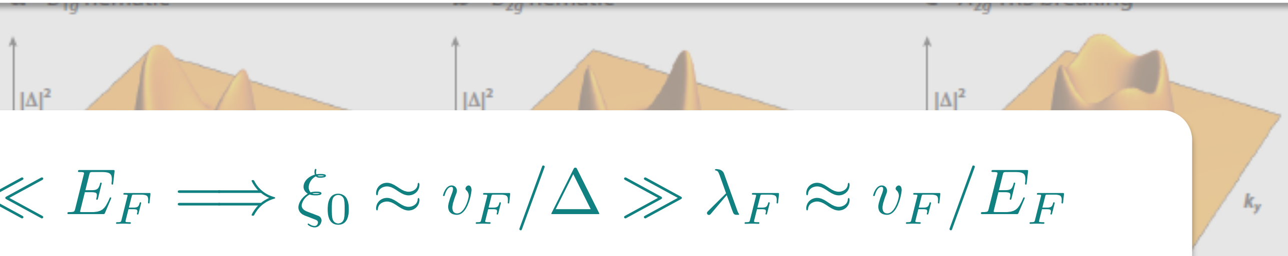
Vestigial phases in superconductors:

Example: p-wave superconductivity in a tetragonal crystal

superconducting order parameter:

$$\langle c_{\mathbf{p}\alpha} c_{-\mathbf{p}\beta} \rangle = \left[\mathbf{d}_{\mathbf{p}} \cdot \boldsymbol{\sigma} i \sigma^{y} \right]_{\alpha\beta} \qquad \mathbf{d}_{\mathbf{p}} = \hat{\mathbf{z}} \left(\Delta_x p_x + \Delta_y p_y \right)$$

Often, superconductors don't fluctuate enough.



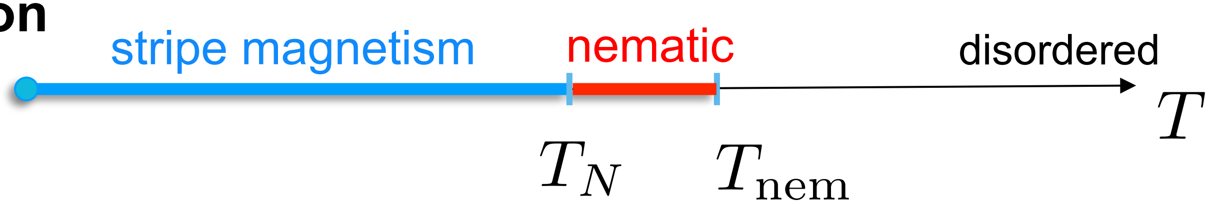
$$\Delta \ll E_F \Longrightarrow \xi_0 \approx v_F/\Delta \gg \lambda_F \approx v_F/E_F$$

Expect vestigial phases in systems with strong fluctuations, like doped Bi_2Se_3 or $MoSe_3$ ...

there is no ordinary second order transition to a multicomponent superconductor

vestigial nematicity in iron-based systems

magnetic fluctuations split the magnetic and nematic phase transition

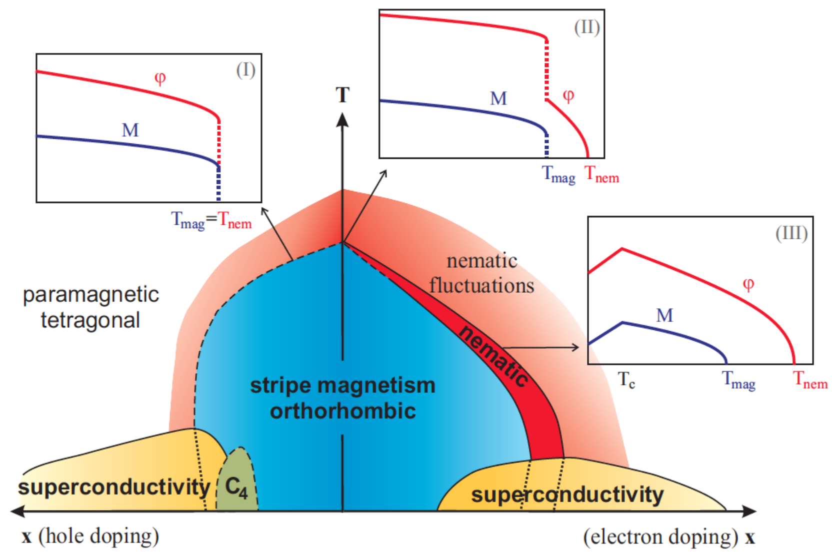


time-reversal symmetry and rotational symmetry are separately broken

P. Chandra, P. Coleman, A. Larkin, PRL (1990),
C. Xu et al. PRB (2008),
C. Fang et al. PRL (2008),
Q. Si and E. Abrahams, PRL (2008)
R. M. Fernandes et al. PRL (2010)

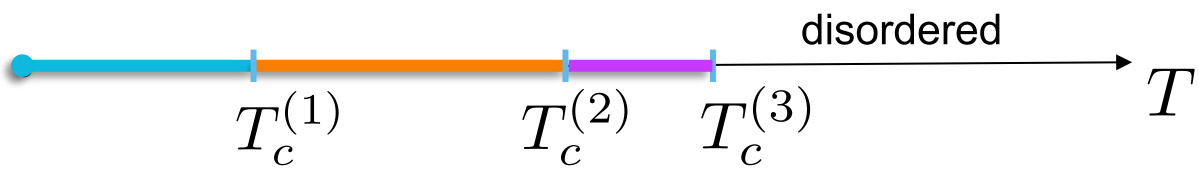
$$\mathbf{S}(\mathbf{r}) = \mathbf{S}_x(\mathbf{r})\, e^{i\mathbf{Q}_x\cdot\mathbf{r}} + \mathbf{S}_y(\mathbf{r})\, e^{i\mathbf{Q}_y\cdot\mathbf{r}}$$

$$\varphi = \langle \mathbf{S}_x^2 - \mathbf{S}_y^2 \rangle$$



R. M. Fernandes, A. Chubukov, and J. S.,
Nature Physics **10**, 97 (2014).

Conclusions:
melting of an order parameter via a cascade of transitions



primary order parameter → composites of the order parameter