Dirac and Weyl fermions:
from Gor'kov equations to Standard Model of particle physics

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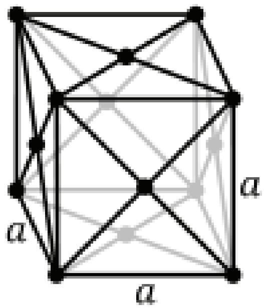
14 Dirac points



28 Weyl points

24 Dirac points
in Standard Model

48 Weyl points
in Standard Model

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for Research & Innovation

Dirac lines, Dirac points & Weyl points in superconductors

Superconducting classes in heavy-fermion systems

G. E. Volovik and L. P. Gor'kov

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
(Submitted 18 October 1984)
Zh. Eksp. Teor. Fiz. **88**, 1412–1428 (April 1985)

TABLE I. Superconductivity classes for one-dimensional representations of the cubic group.

Representation		Class	Heat capacity $C_e(T)$
A_1	$S=0,\ A_{1g}$	$O\times R$	$\exp(-\Delta/T)$
	$S=1,\ A_{1u}$		$\exp(-\Delta/T)$
A_2	$S=0,\ A_{2g}$	$O(T)\times R$	T^2
	$S=1,\ A_{2u}$		T^3

Dirac lines
Dirac points

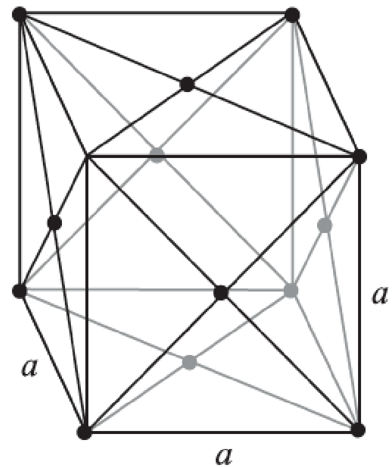
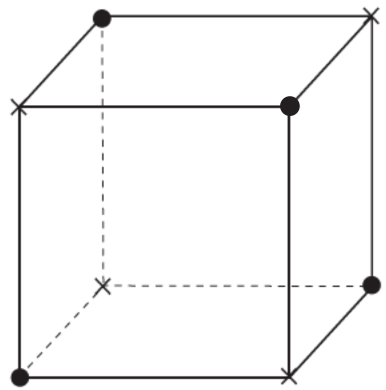


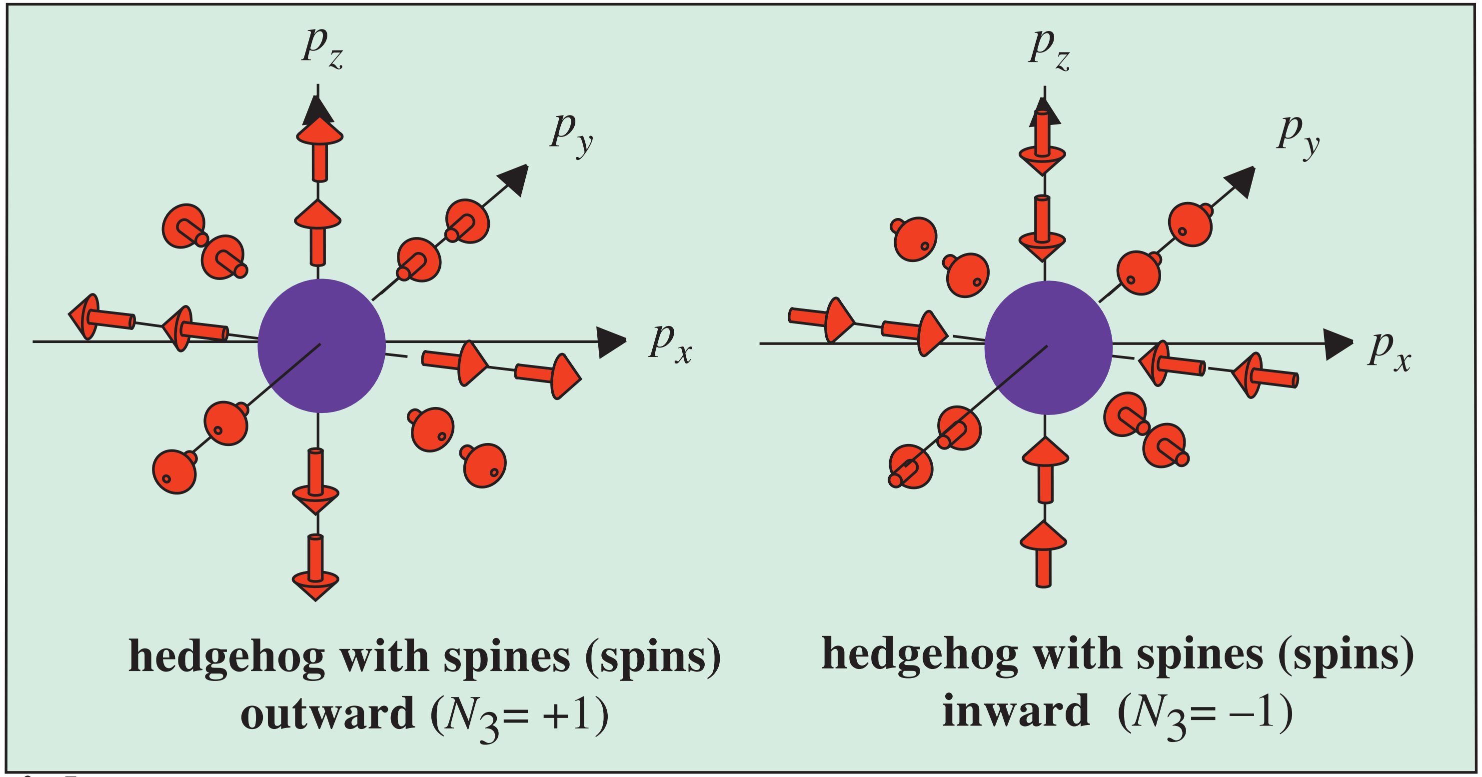
TABLE II. Superconductivity classes from two-dimensional representations E of the cubic group.

(η_1, η_2)		Class	$C_e(T)$	Degeneracy	Magn. properties
(1, 0)	$S=0,\ E_g$	$O(D_2)$	T^3	2	A
	$S=1,\ E_u$		T^3	2	A
(1, −1)	$S=0,\ E_g$	$D_4^{(1)}(D_2)\times R$	T^2	3	–
	$S=1,\ E_u$		T^3	3	–
(1, 1)	$S=0,\ E_g$	$D_4\times R$	$\exp(-\Delta/T)$	3	–
	$S=1,\ E_u$		$\exp(-\Delta/T)$	3	–

Weyl points
Weyl points
Dirac lines
Dirac points

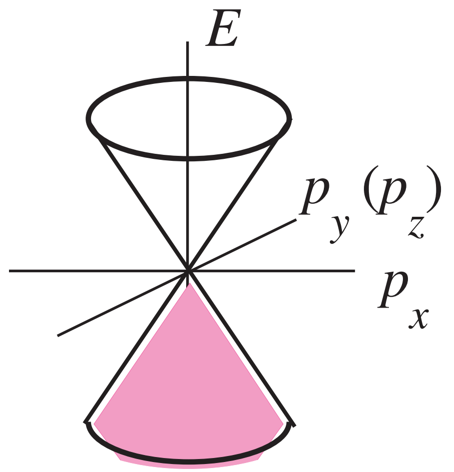


Right-handed & left-handed Weyl particles



right $H = +c\,\sigma \cdot \mathbf{p}$ $\quad$ $H = -c\,\sigma \cdot \mathbf{p}$ left

$$E^2 = c^2 p^2$$

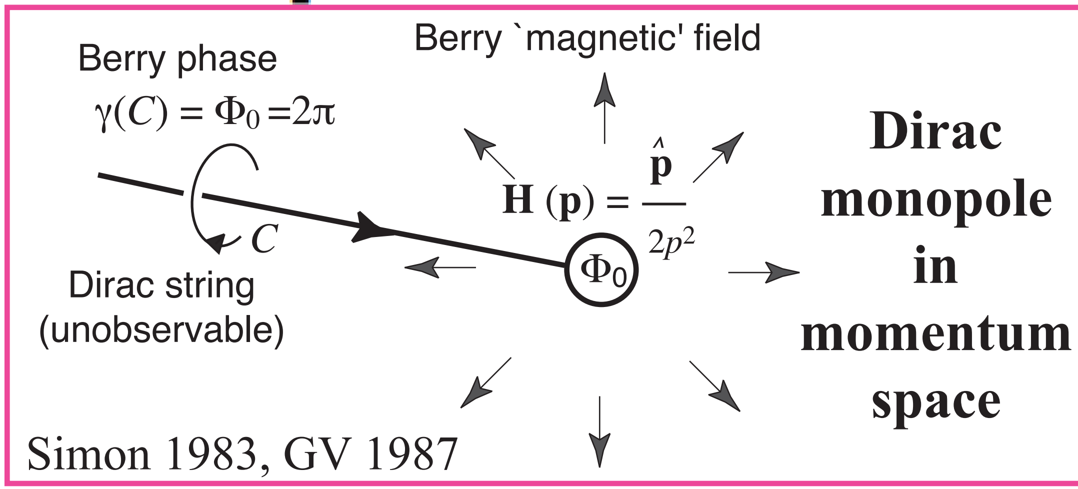


Weyl point:
conical (diabolical)
crossing point
in fermionic spectrum
in momentum space

boojums
(GV, Mineev 1982)

$$N = \frac{e_{\alpha\beta\mu\nu}}{24\pi^2}\ \mathbf{tr}\left[\int_\sigma dS^\alpha\ G\partial_{p_\beta}G^{-1}G\partial_{p_\mu}G^{-1}G\partial_{p_\nu}G^{-1}\right]$$

in superconductors & chiral 3He-A
topological invariant for Wel points
is expressed in terms of
Gor'kov Green's function (1958)



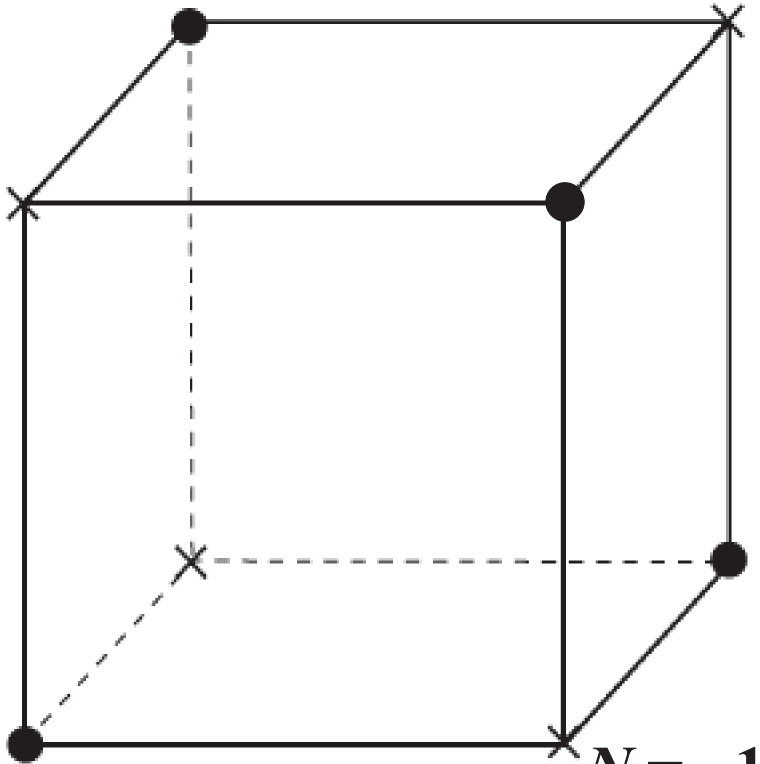
8 Weyl fermions in superconductors with broken time reversal symmetry: class $O(D_2)$

$$N = \frac{e_{\alpha\beta\mu\nu}}{24\pi^2}\ \mathbf{tr}\left[\int_\sigma dS^\alpha\ G\partial_{p_\beta}G^{-1}G\partial_{p_\mu}G^{-1}G\partial_{p_\nu}G^{-1}\right]$$

topological invariant for Wel points in terms of Gor'kov Green's function

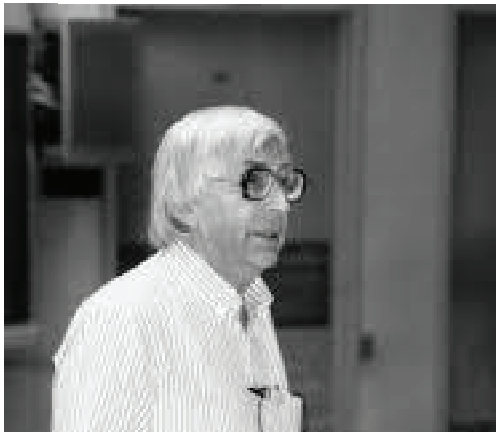
G = 1/(iω - H)

$$H = \epsilon(\mathrm{p})\tau_3 + a\frac{1}{2}(2p_x^2 - p_y^2 - p_z^2)\tau_1 + a\frac{\sqrt{3}}{2}(p_y^2 - p_z^2)\tau_2$$



$N = +1$
right quasiparticle

$N = -1$
left quasiparticle

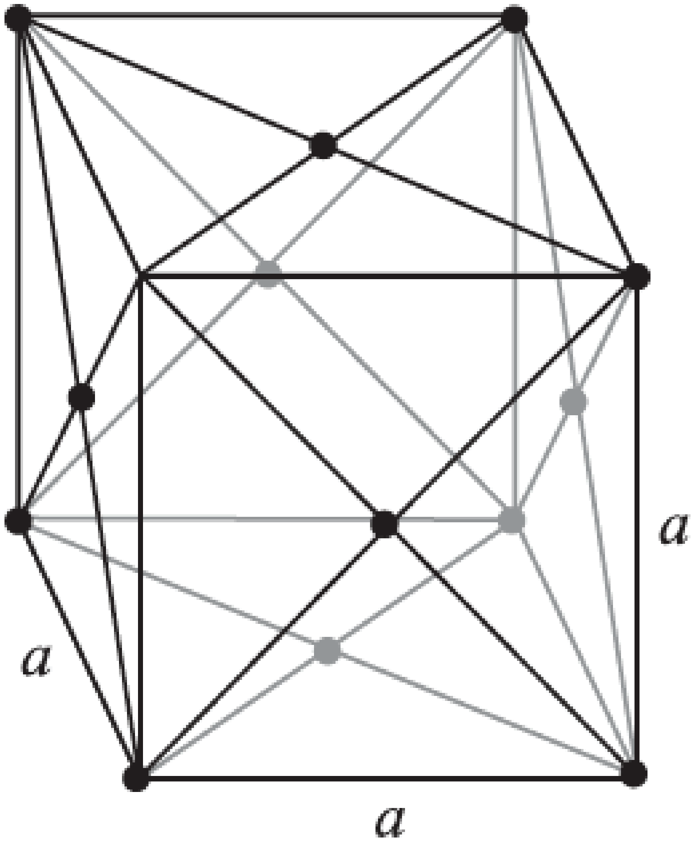


Superconducting classes in heavy-fermion systems

G. E. Volovik and L. P. Gor'kov 1985

14 Dirac fermions in superconductors with time reversal symmetry: class O(T) x R

$$N = \frac{e_{\alpha\beta\mu\nu}}{24\pi^2} \mathbf{tr} \left[\tau_2 \int_\sigma dS^\alpha \; G\partial_{p_\beta} G^{-1} G\partial_{p_\mu} G^{-1} G\partial_{p_\nu} G^{-1} \right]$$



topological invariant for Dirac points in terms of Gor'kov Green's functions

$$\hat{G}^{-1} = i\omega + \tau_3 \epsilon(\mathbf{p}) + \tau_1 \sigma_i d_i(\mathbf{p}),$$

where the vector gap function is

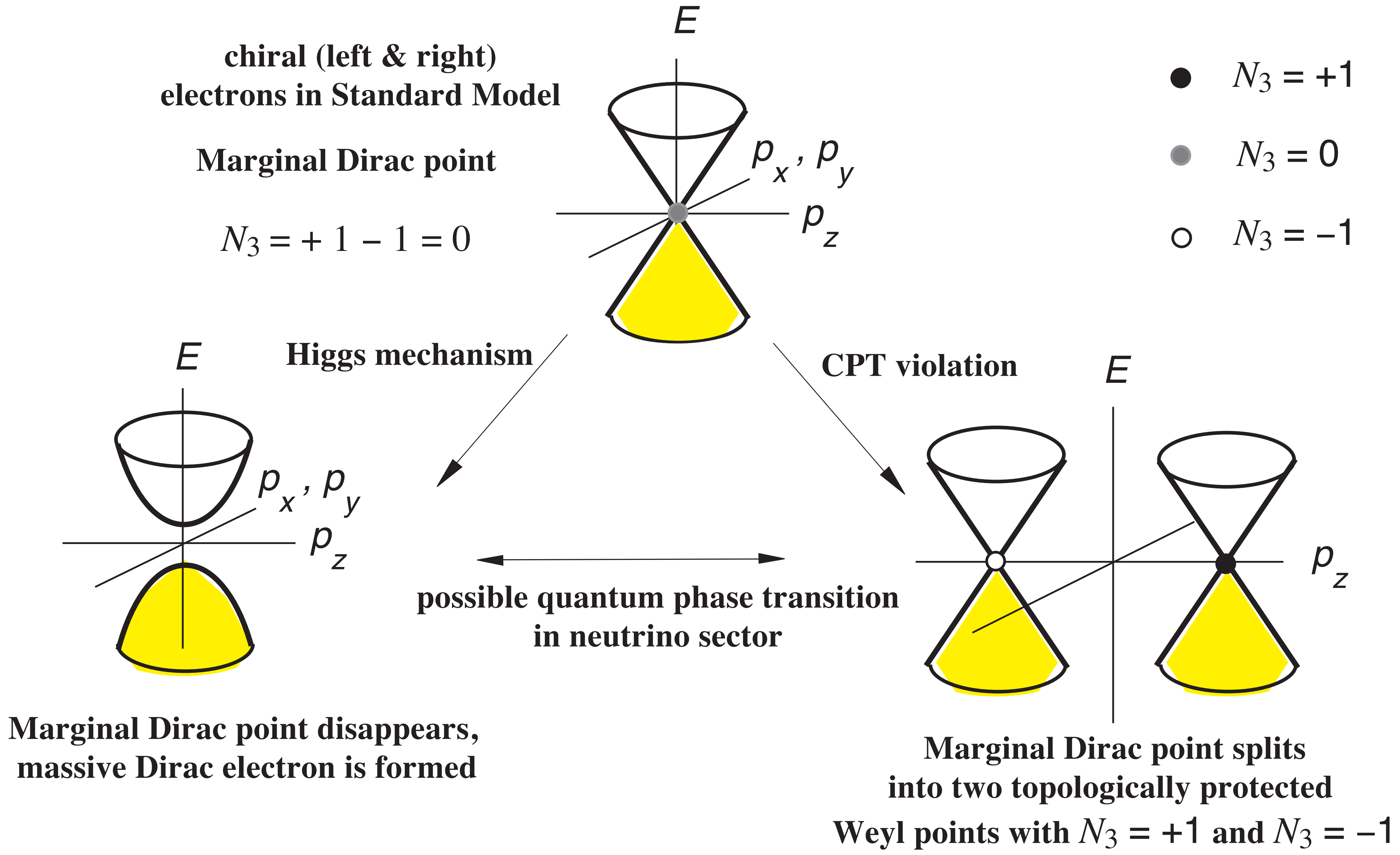
$$\mathbf{d}(\mathbf{p}) \propto \hat{\mathbf{x}} p_x (p_y^2 - p_z^2) + \hat{\mathbf{y}} p_y (p_z^2 - p_x^2) + \hat{\mathbf{z}} p_z (p_x^2 - p_y^2).$$

14 Dirac fermions split to 28 Weyl fermions when time reversal symmetry is violated

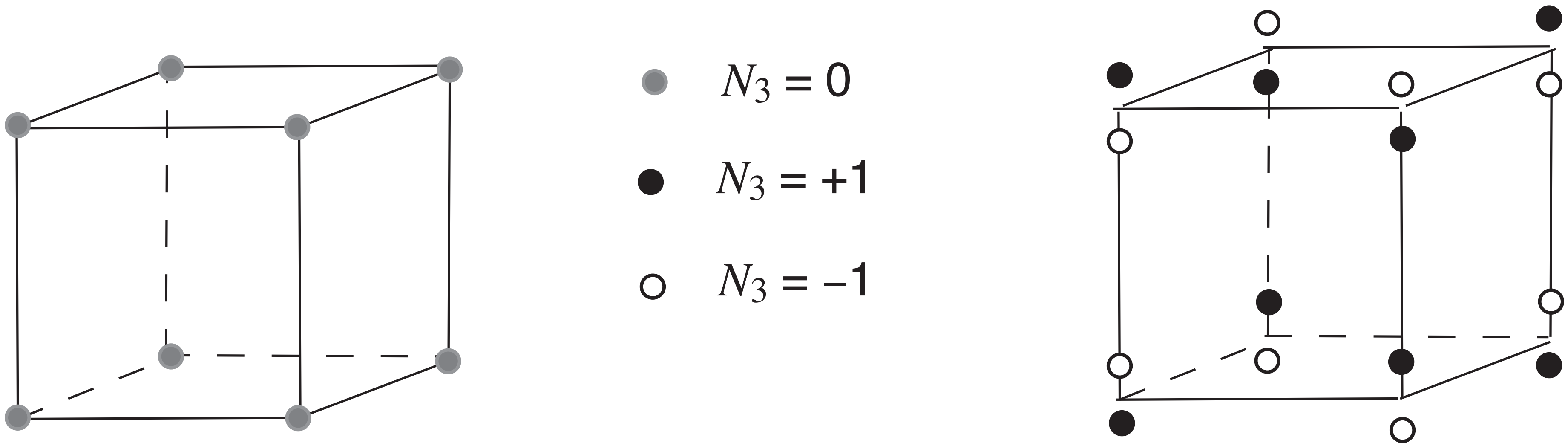
Superconducting classes in heavy-fermion systems

G. E. Volovik and L. P. Gor'kov 1985

Two topological scenaria in Standard Model

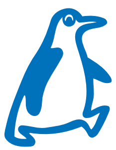


splitting of Dirac points to pairs of Weyl points by breaking of time reversal symmetry



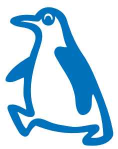
8 Dirac points

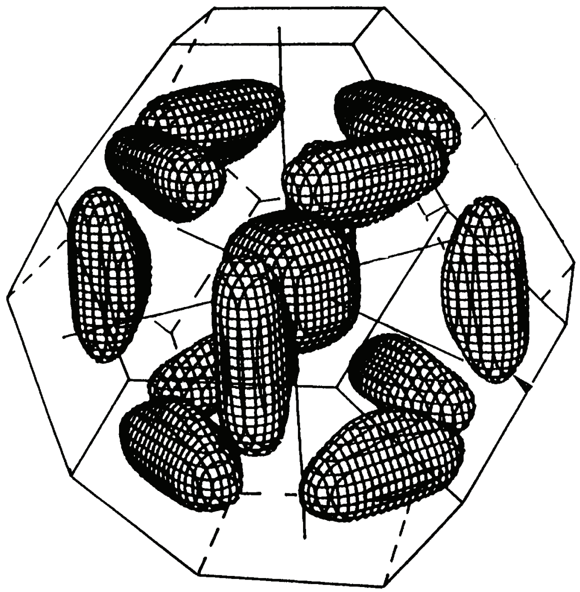
16 Weyl points



"Standard Model" with
8 left-handed $N_3 = -1$
chiral fermions

and 8 right-handed $N_3 = +1$
chiral fermions





Fermi pockets may contain large number of Weyl & Dirac points

Agterberg, Barzykin, Gor'kov (1999)

Dirac lines in cuprates

Superconducting classes in heavy-fermion systems

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TABLE VII. Superconductivity classes from one-dimensional representations of group D_4.

Representation	Type of basis function	Class	$C_e(T)$	
A_1 $S=0,\ A_{1g}$	Symmetr. Function	$D_4 \times R$	$\exp(-\Delta/T)$	
A_1 $S=1,\ A_{1u}$	$a\tilde{\mathbf{z}}k_z + b(\tilde{\mathbf{x}}k_x + \tilde{\mathbf{y}}k_y)$		$\exp(-\Delta/T)$	
A_2 $S=0,\ A_{2g}$	$k_xk_y(k_x^2-k_y^2)$	$D_4(C_4)\times R$	T^2	
A_2 $S=1,\ A_{2u}$	$(\tilde{\mathbf{x}}k_y+\tilde{\mathbf{y}}k_x)(k_x^2-k_y^2)$		T^3	
B_1 $S=0,\ B_{1g}$	$k_x^2-k_y^2$	$D_4^{(1)}(D_2)\times R$	T^2	Dirac lines
B_1 $S=1,\ B_{1u}$	$\tilde{\mathbf{x}}k_x-\tilde{\mathbf{y}}k_y$		T^3	
B_2 $S=0,\ B_{2g}$	k_xk_y	$D_4^{(2)}(D_2)\times R$	T^2	Dirac lines
B_2 $S=1,\ B_{2u}$	$\tilde{\mathbf{x}}k_y+\tilde{\mathbf{y}}k_x$		T^3	

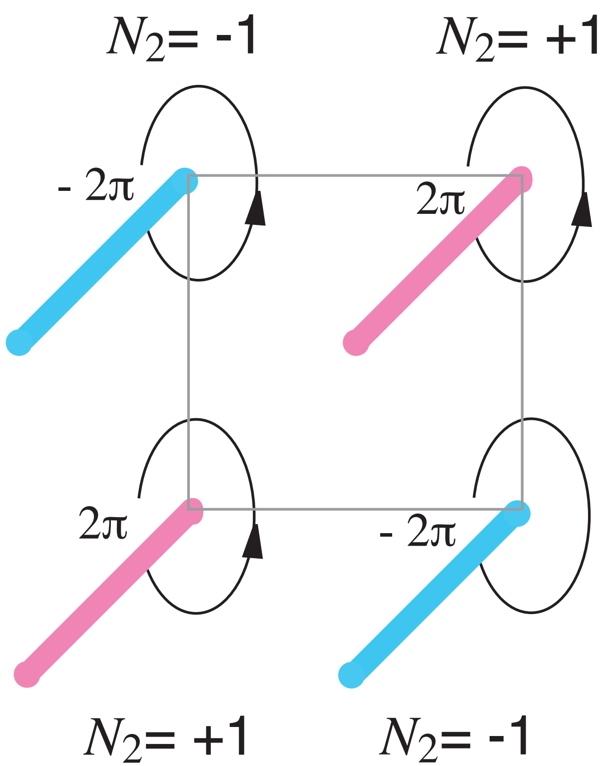
Dirac nodal lines in *cuprate* superconductors

$$H = \begin{pmatrix} \frac{p^2}{2m} - \mu & a(p_x^2 - p_y^2) \\ a(p_x^2 - p_y^2) & \mu - \frac{p^2}{2m} \end{pmatrix}$$

$$H(\mathbf{p}) = C(\tau_3 \cos\phi(\mathbf{p}) + \tau_1 \sin\phi(\mathbf{p}))$$

N_2 -- winding number of $\phi(\mathbf{p})$ around nodal line

4 nodal lines

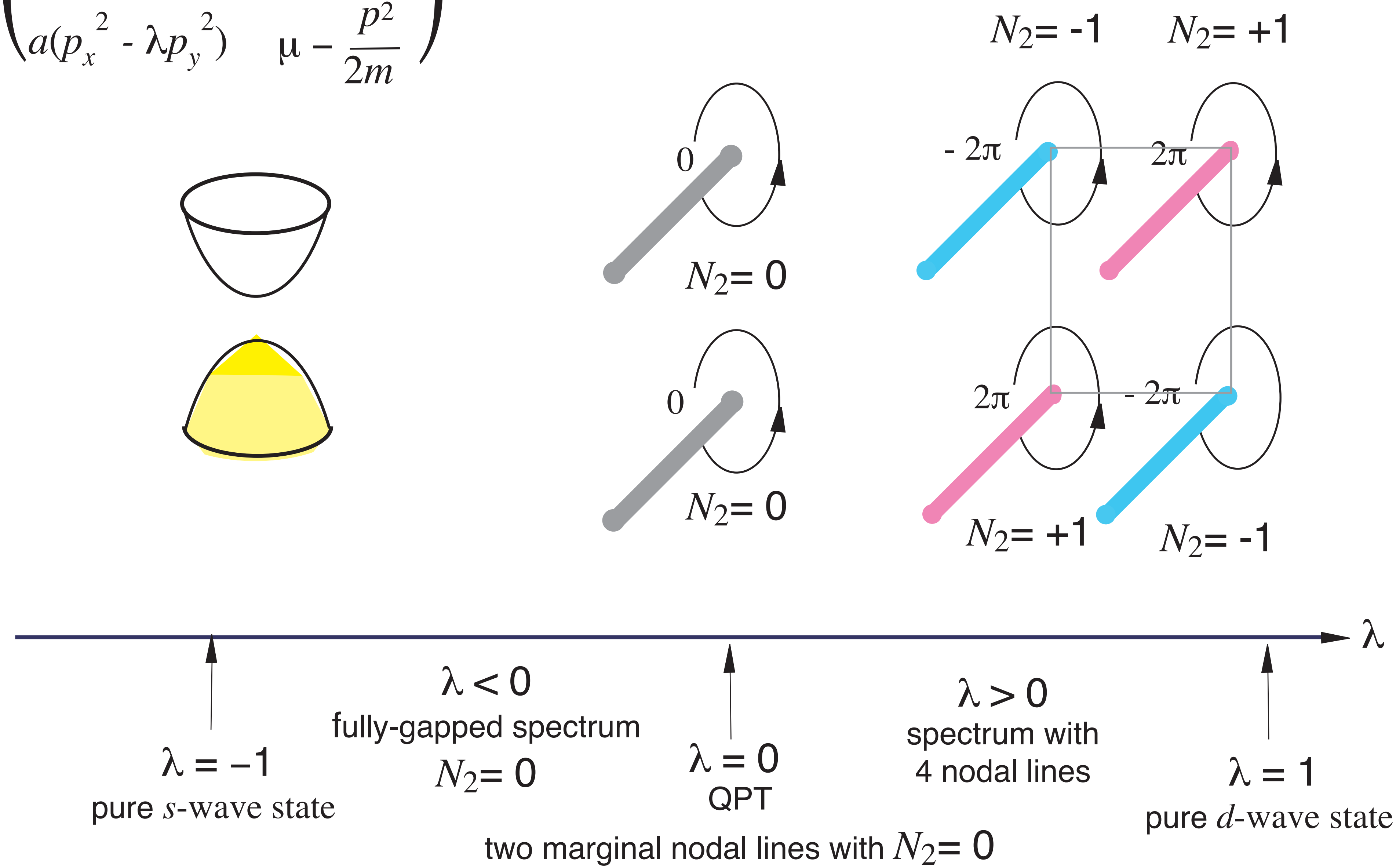


Nodal line transforms to Bogoliubov Fermi surface under supercurrent

Topological quantum phase transition:

*** s+d high-T_c superconductors** $Pr_{2-x}Ce_xCuO_{4-\delta}$???

$$H = \begin{pmatrix} \frac{p^2}{2m} - \mu & a(p_x^2 - \lambda p_y^2) \\ a(p_x^2 - \lambda p_y^2) & \mu - \frac{p^2}{2m} \end{pmatrix} = C(\tau_3 \cos\phi(\mathbf{p}) + \tau_1 \sin\phi(\mathbf{p}))$$

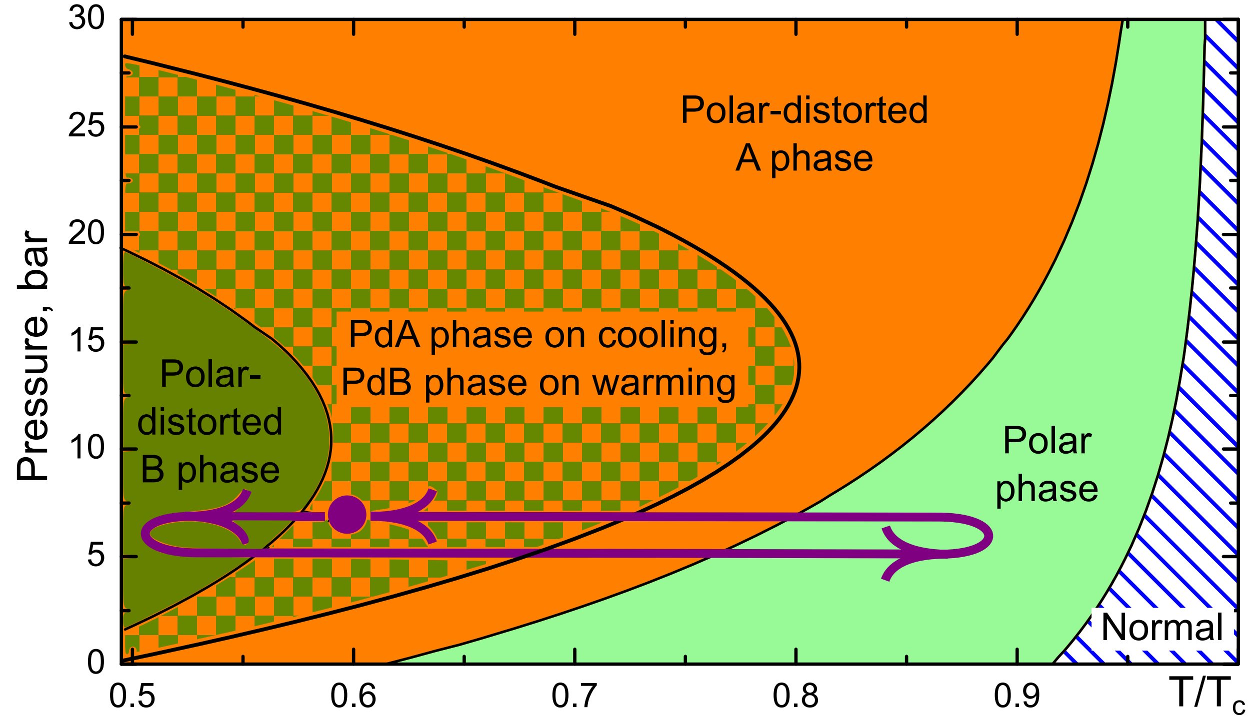


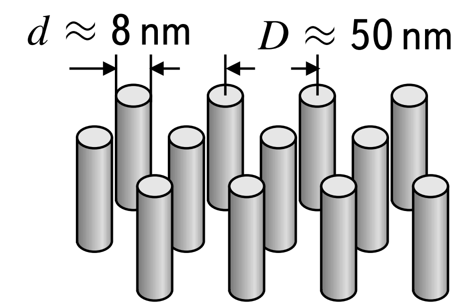
Sauls, Halperin, Parpia

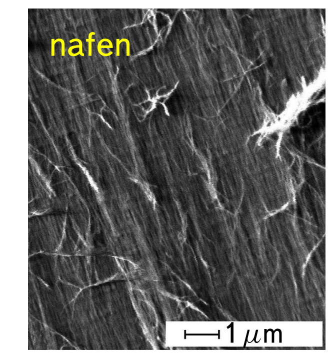
Superfluid ^{3}He in aerogel confinement

Aoyama, Ikeda 2006
Dmitriev et al. 2015
ROTA group, AALTO

When superfluid ^{3}He is confined to anisotropic aerogel (”nafen”), a new phases stabilize

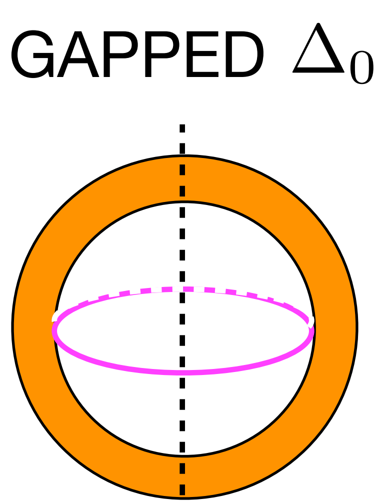






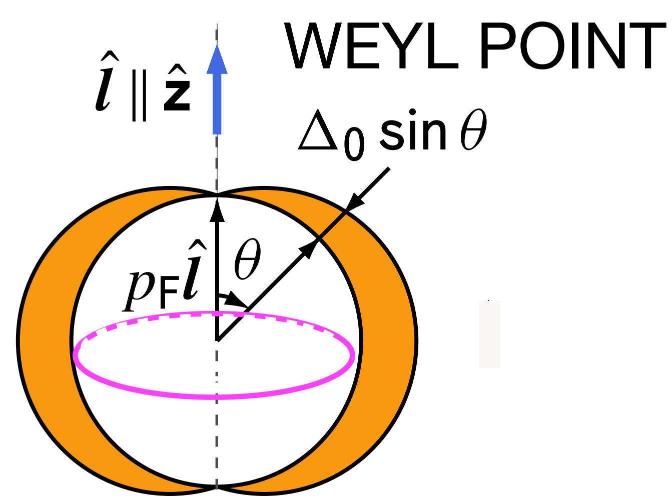
polar T$_c$ suppression: in secular scattering $k_{||}$ conserved; Anderson theorem applies

Fomin 2018



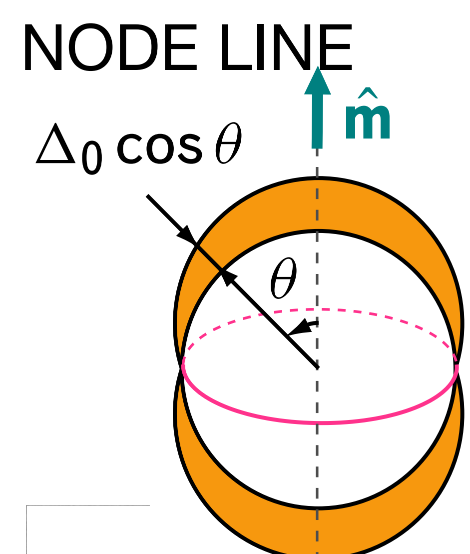
B-phase

$$A_{\mu i} = \Delta_0 e^{i\Phi}\delta_{\mu i}$$



A-phase

$$A_{\mu i} = \Delta_0 e^{i\Phi}\hat{d}_\mu(\hat{m}_i + i\hat{n}_i)$$



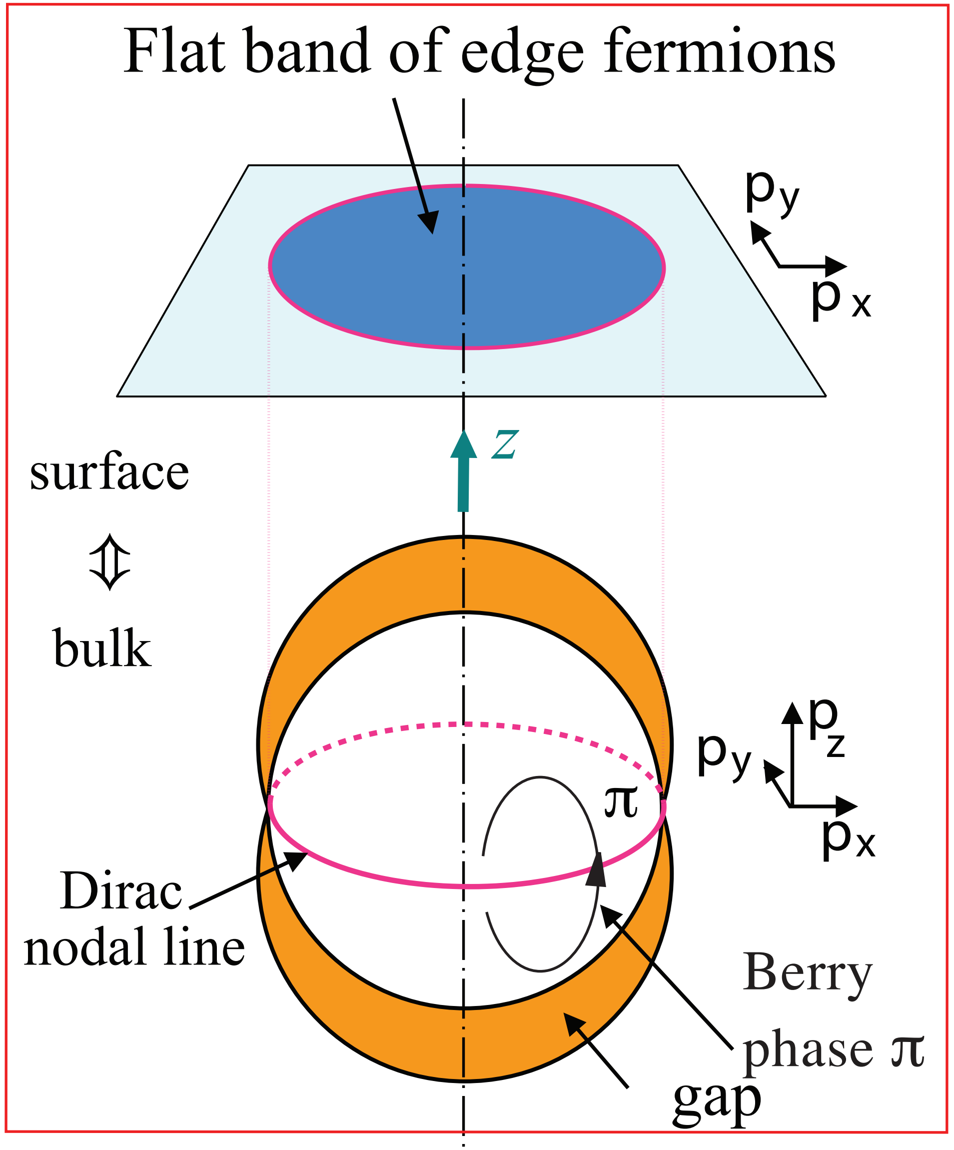
Polar phase

$$A_{\mu i} = \Delta_0 e^{i\Phi}\hat{d}_\mu\hat{m}_i$$

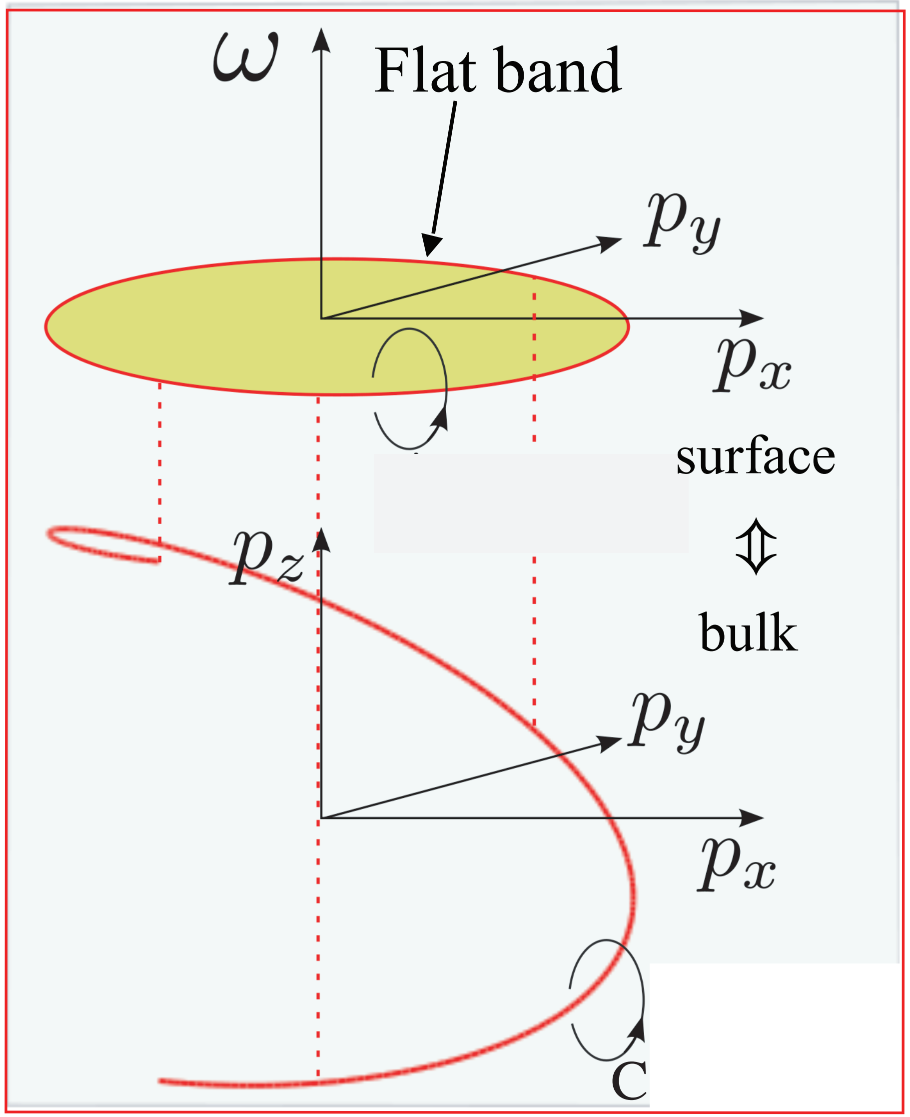
Dirac nodal line generates flat band on the surface

projection of nodal line on the surface determines boundary of 2D flat band

polar phase of 3He



rhombohedral graphite



Berry phase = π

Flat band - route to room-T superconductivity

Khodel-Shaginyan, JETP Lett. **51**, 553 (1990)
GV, JETP Lett. **53**, 222 (1991)
Nozieres, J. Phys. (Fr.) **2**, 443 (1992)

$$\Delta = g \int \frac{d^3p}{2h^3} \frac{\Delta}{E(p)}$$

BCS gap equation

g - coupling in Cooper channel

$$E^2(p) = \Delta^2 + \varepsilon^2(p)$$

metal with **Fermi surface**

$$\varepsilon(p) = v_F\,(p - p_F)$$

$$E^2(p) = \Delta^2 + v_F^{\,2}(p - p_F)^2$$

$$1 = gN(0) \int \frac{d\varepsilon}{E(\varepsilon)} = gN(0) \ln \frac{E_c}{\Delta}$$

$$T_c \sim \Delta = E_c \exp\,[-1/gN(0)]$$

typical superconductivity:
exponentially suppressed
transition temperature

metal with **flat band**

$$\varepsilon(p) = 0$$

in flat band

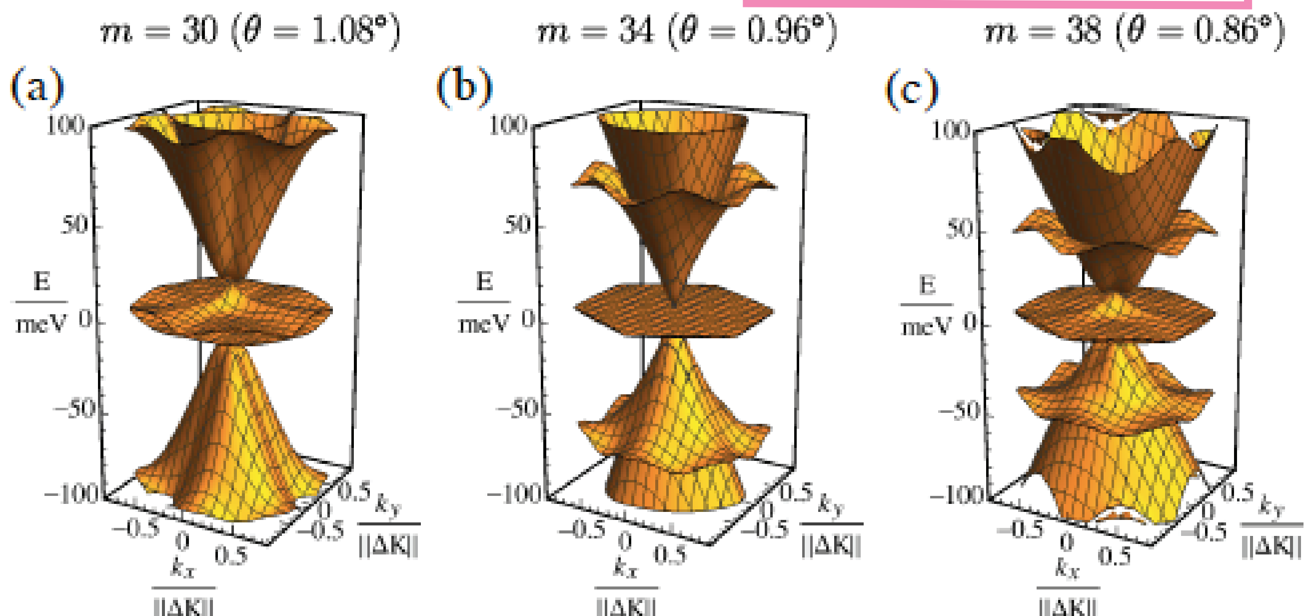
$$E(p) = \Delta$$

$$\Delta = g \int \frac{d^3p}{2h^3} = gV_{FB}$$

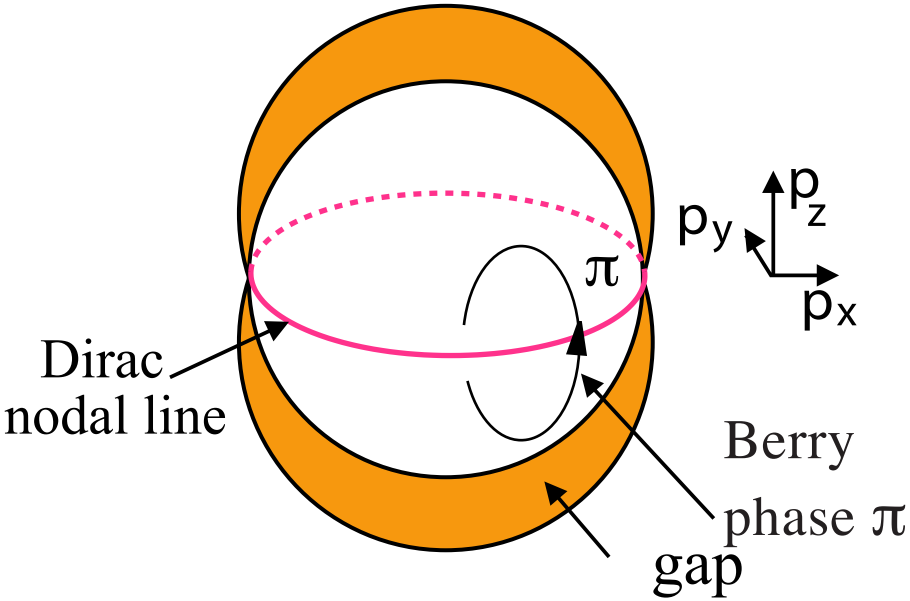
flat band volume

$$T_c \sim \Delta = gV_{FB}$$

flat band superconductiv
linear dependence
of T_c on coupling g



Super-Landau superlow:
Bogolliubov Fermi surface in Dirac superfluids & superconductors

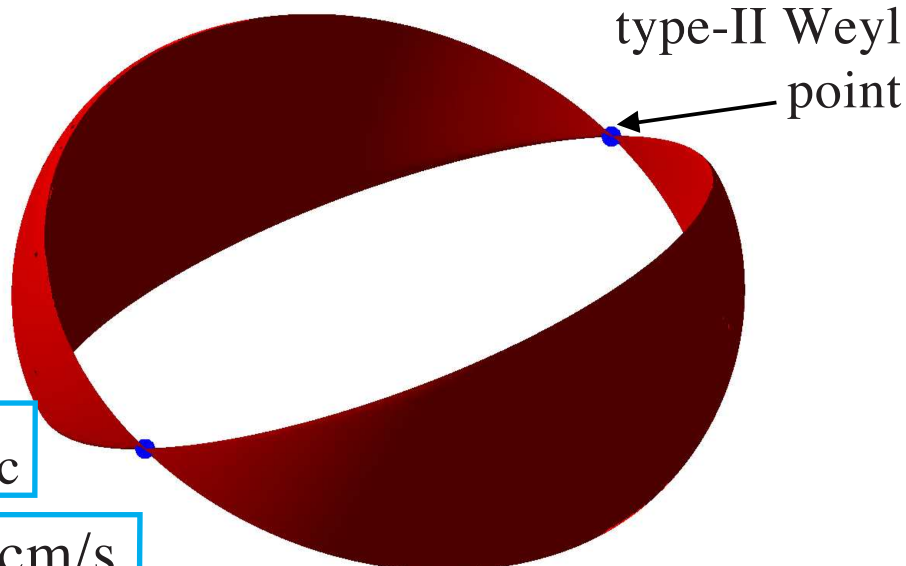


$v_{\text{Landau}} = 0$

$$H = \begin{pmatrix} \frac{p^2}{2m} - \mu & cp_z \\ cp_z & -\frac{p^2}{2m} + \mu \end{pmatrix} + \mathbf{p}\cdot\mathbf{v}_S$$

Doppler shift

Supercritical superflow in polar phase



$0 < v < v_c$

$v_c = 0.2$ cm/s
vortex creation velocity

Dirac line expands to two Bogoliubov Fermi pockets

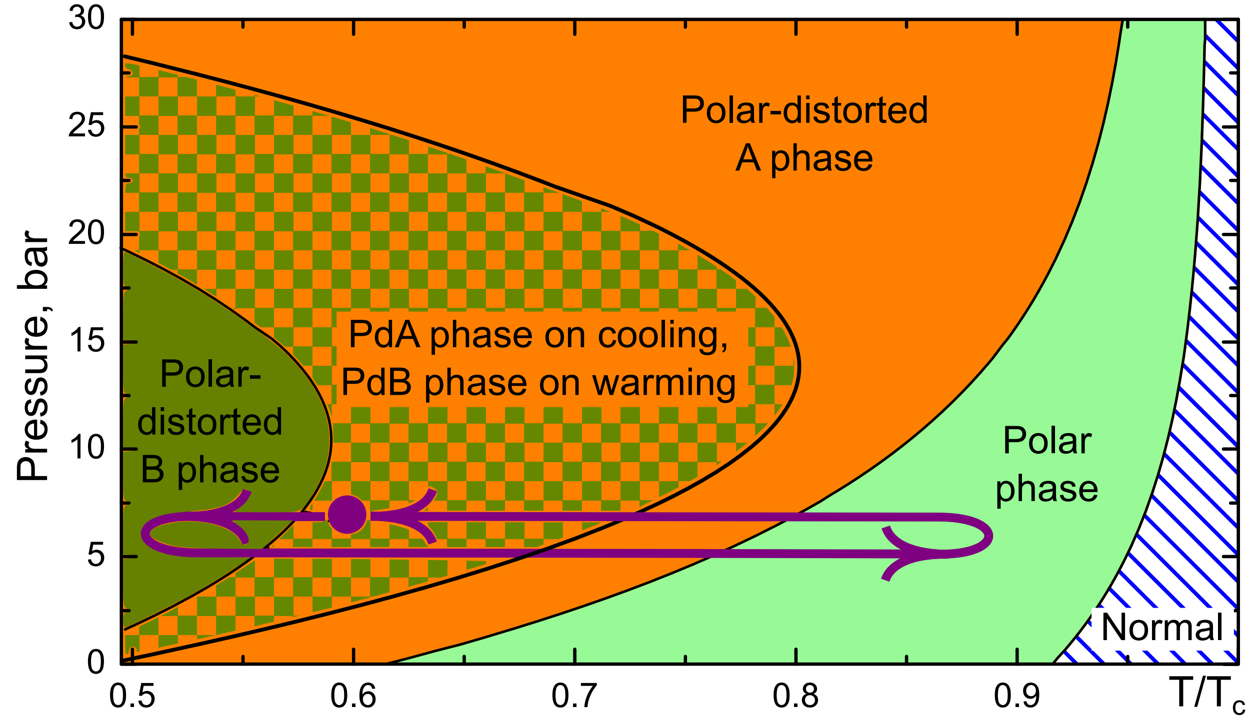
cuprate superconductors also contain
flat edge modes
&
Bogolliubov Fermi surfaces:
$\mathbf{B}^{1/2}$ density of states (GV 1993)
корешок (Gor'kov)

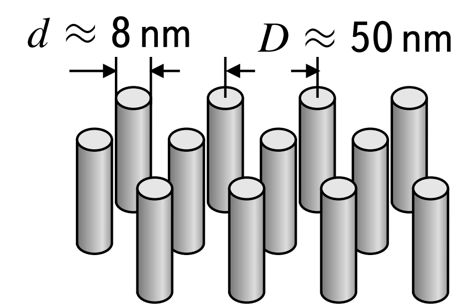
Sauls, Halperin, Parpia

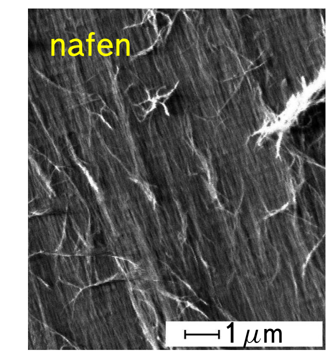
Superfluid ^{3}He in aerogel confinement

Aoyama, Ikeda 2006
Dmitriev et al. 2015
ROTA group, AALTO

When superfluid ^{3}He is confined to anisotropic aerogel ("nafen"), a new phases stabilize

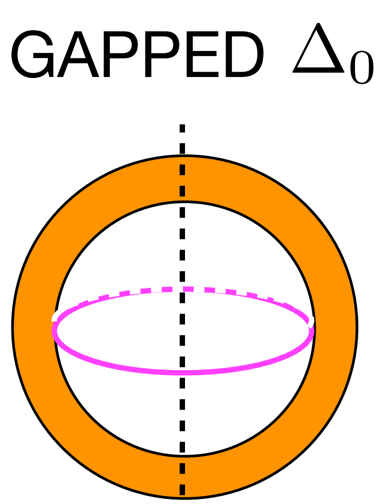






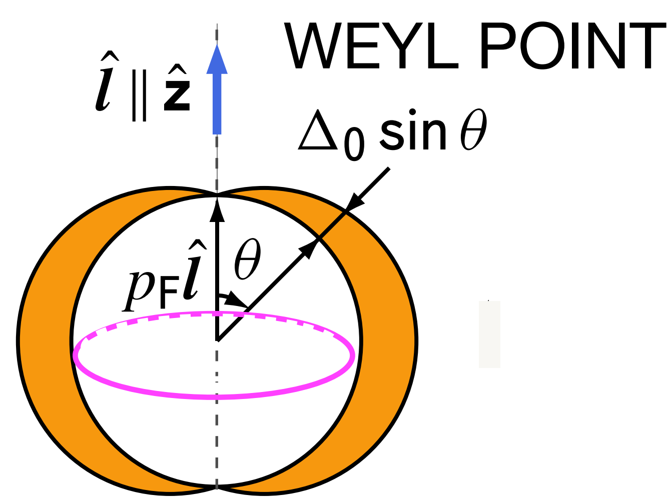
polar T_c suppression: in secular scattering $k_{||}$ conserved; Anderson theorem applies

Fomin 2018



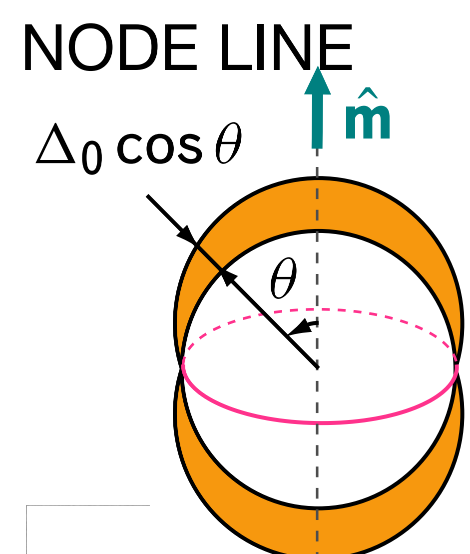
B-phase

$$A_{\mu i} = \Delta_0 e^{i\Phi} \delta_{\mu i}$$



A-phase

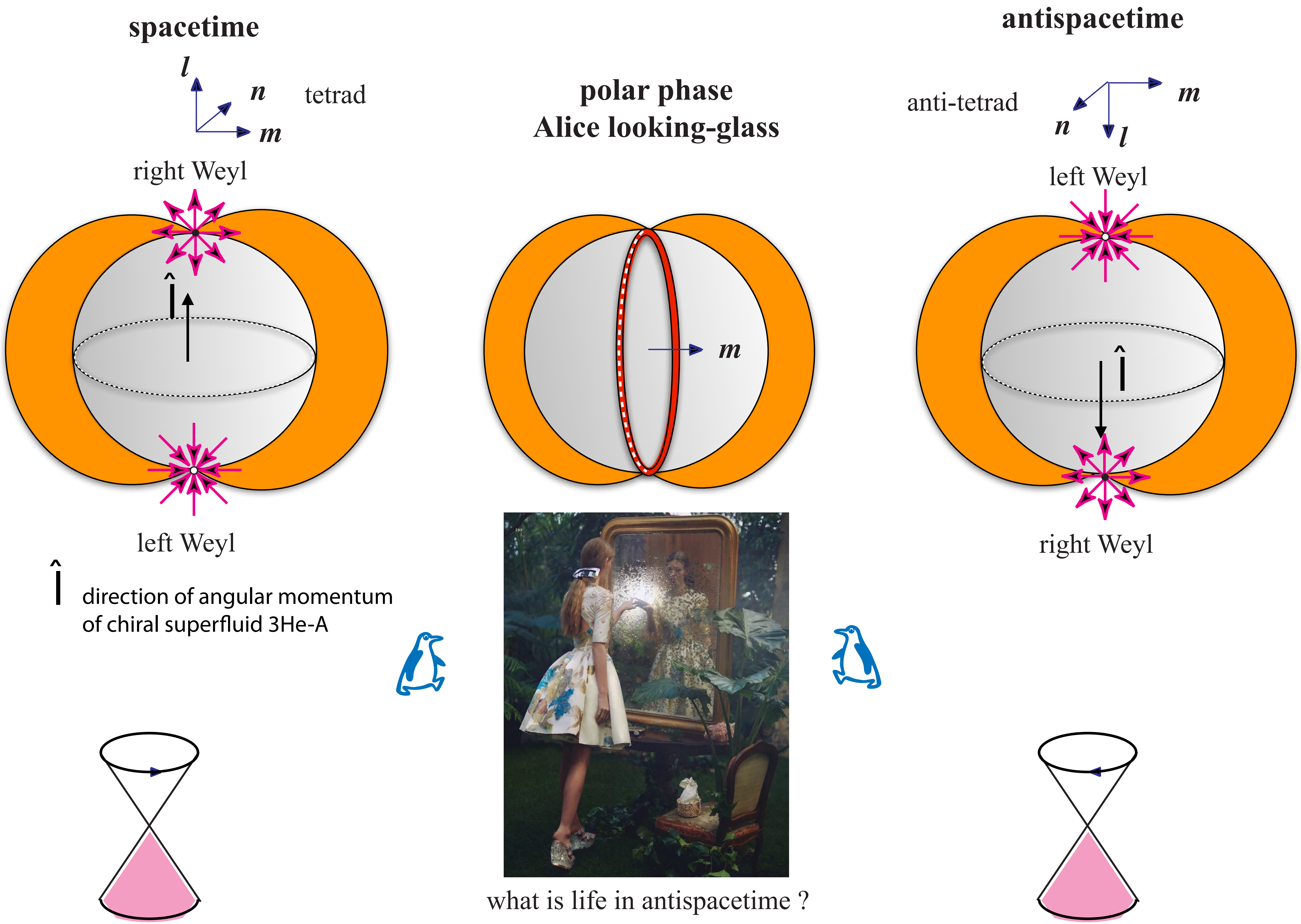
$$A_{\mu i} = \Delta_0 e^{i\Phi} \hat{d}_\mu (\hat{m}_i + i\hat{n}_i)$$



Polar phase

$$A_{\mu i} = \Delta_0 e^{i\Phi} \hat{d}_\mu \hat{m}_i$$

From Weyl to nodal line and anti-Weyl: from spacetime to antispacetime



what is life in antispacetime ?

QED in spacetime and antispacetime

$$L_{\rm em} = \frac{\sqrt{-g}}{24\pi^2} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} \ln\left(\frac{E_{\rm UV}}{E_{\rm IR}}\right)$$

$$-g = (\det\, e)^2$$

what happens when **det e** changes sign and **spacetime** transforms to **anti-spacetime** ?

$$L_{\rm em} = \frac{\det\, e}{24\pi^2} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} \ln\left(\frac{E_{\rm UV}}{E_{\rm IR}}\right)$$

analytic

or

$$L_{\rm em} = \frac{|\det\, e|}{24\pi^2} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} \ln\left(\frac{E_{\rm UV}}{E_{\rm IR}}\right)$$

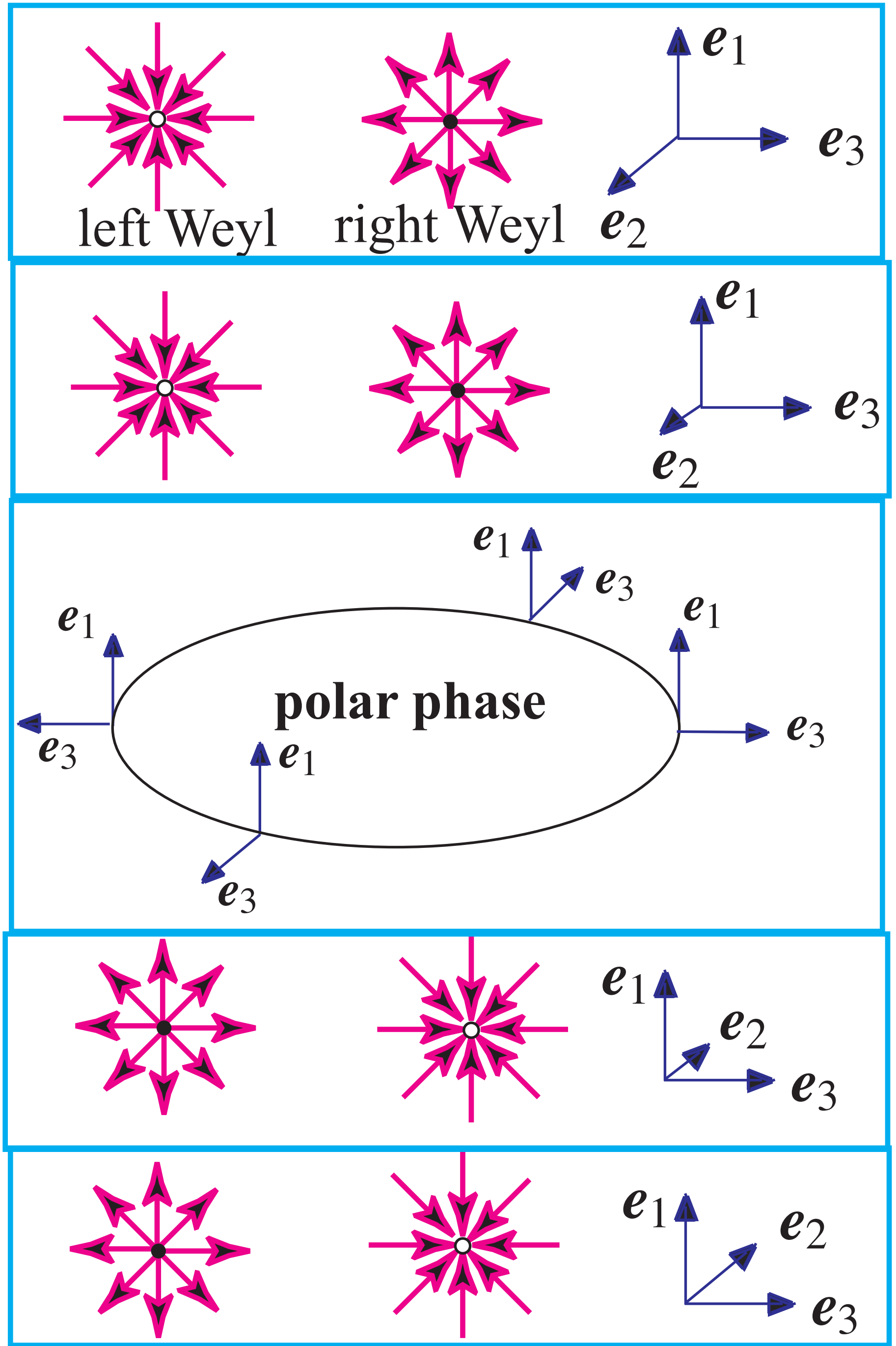
non-analytic

Diakonov 2011

Rovelli 2012

transition to anti-spacetime is non-analytic

spacetime

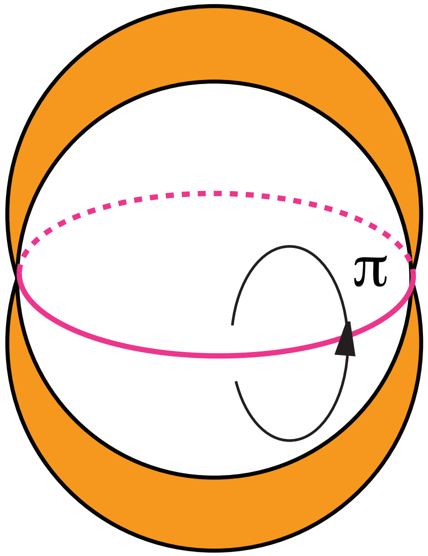


anti-spacetime

A-phase

$$L_{\rm em} = \frac{|\det e|}{24\pi^2} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} \ln\left(\frac{E_{\rm UV}}{E_{\rm IR}}\right)$$

polar-distorted A-phase



intermediate degenerate spacetime with $e_2 = 0$

$$L^{2+1}[B] = \sqrt{g_\perp}\frac{\zeta(3/2)}{4\sqrt{2}\pi^2}|B|^{3/2}$$

polar phase

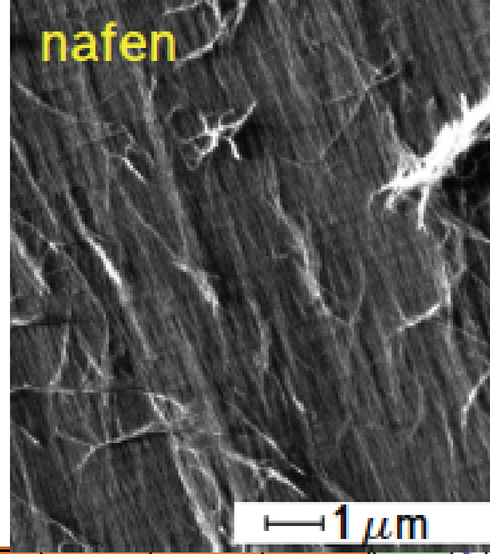
polar-distorted A-phase

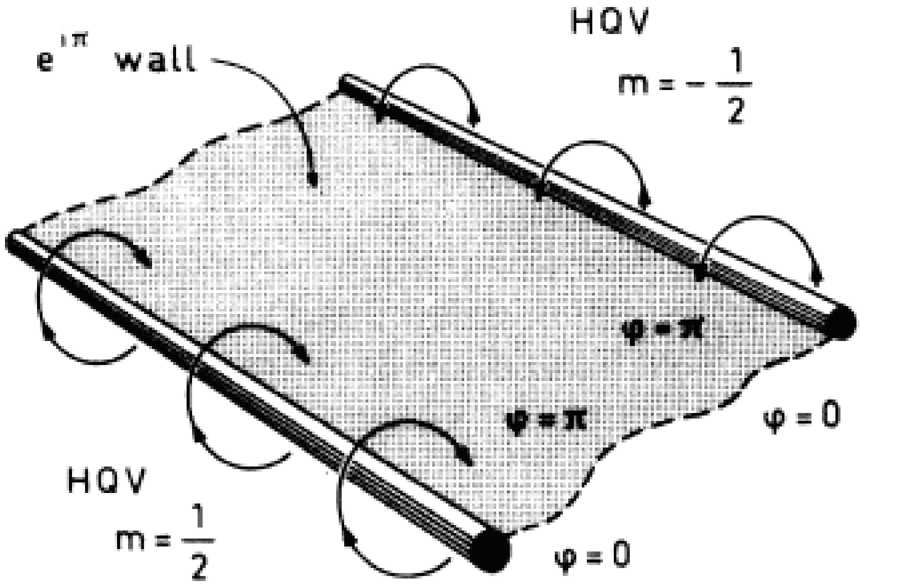
Nissinen, GV 2018

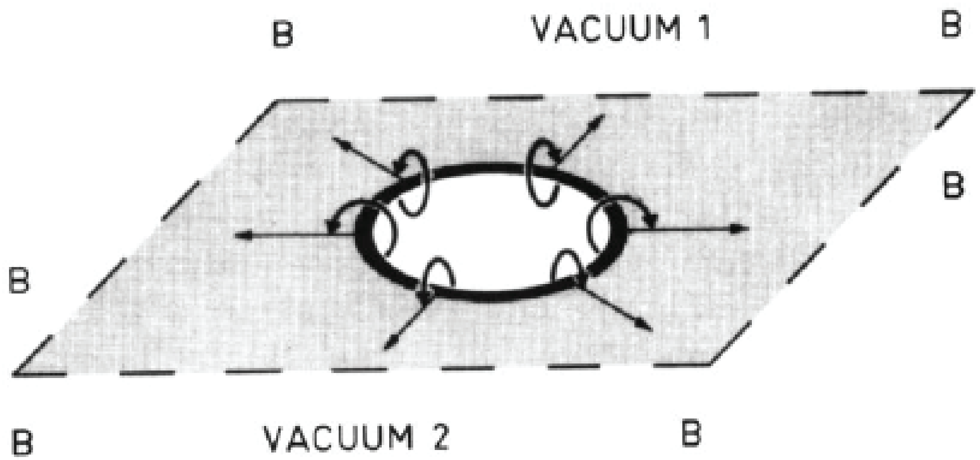
A-phase

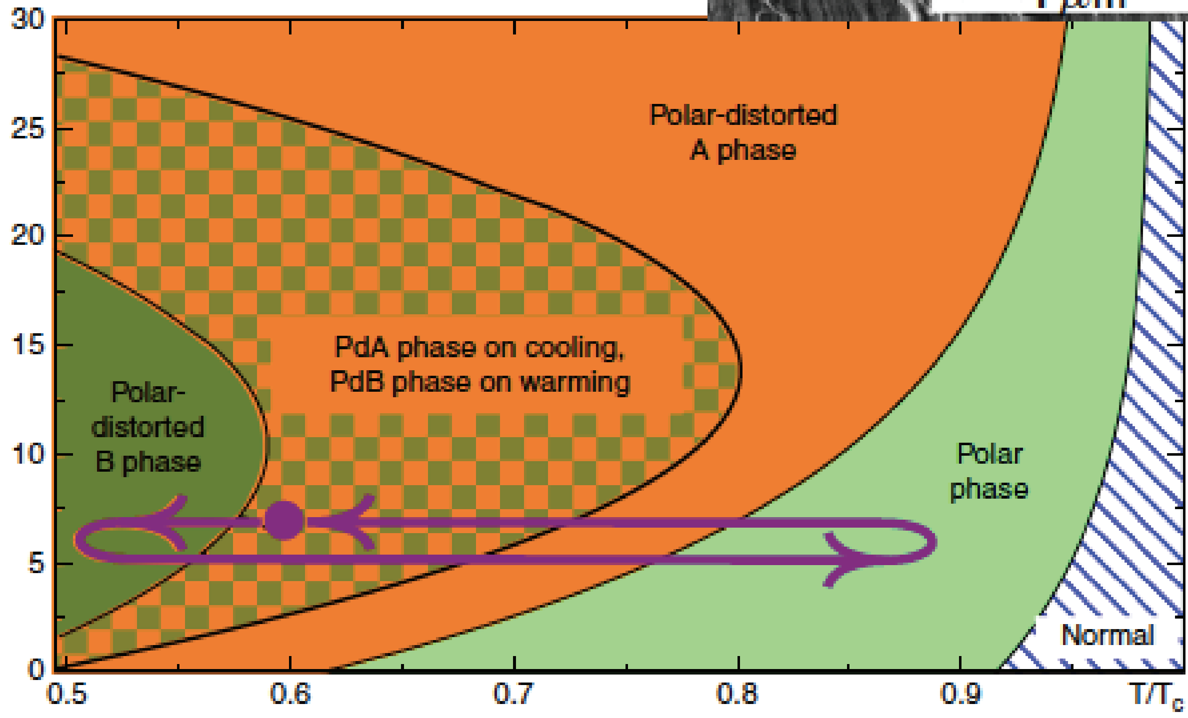
$$L_{\rm em} = \frac{|\det e|}{24\pi^2} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} \ln\left(\frac{E_{\rm UV}}{E_{\rm IR}}\right)$$

Kibble-Lazarides-Shafi walls bounded by strings (1982)





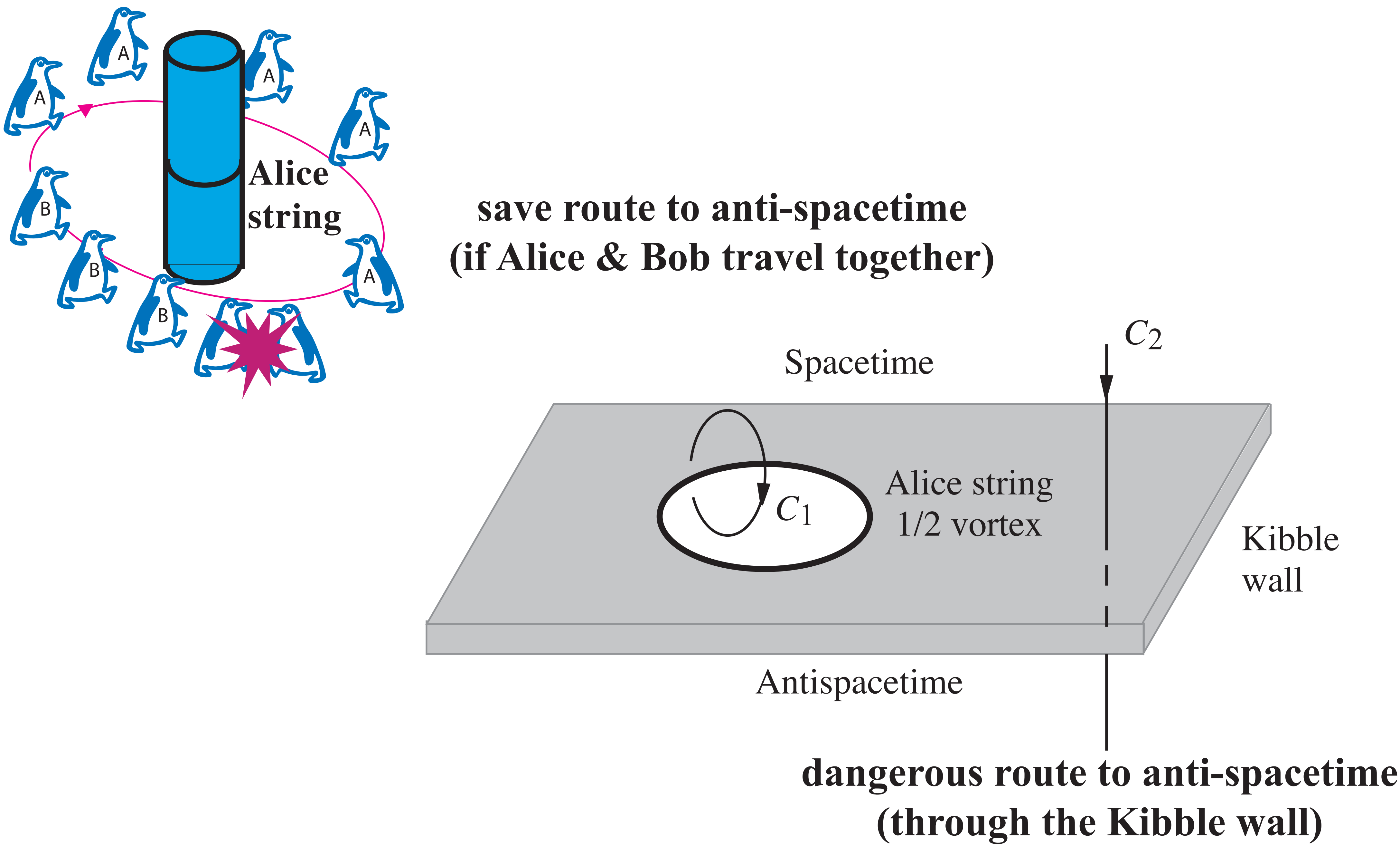




Cosmiclike domain walls in superfluid 3He-B
Salomaa, GV (1988)

Mäkinen, Dmitriev, Nissinen, Rysti, GV, Yudin, Zhang, Eltsov
Half-quantum vortices and walls bounded by strings in the polar-distorted phasesof topological superfluid 3He
Nat. Comm. **10,** 237 **(2019)**

save and dangerous transitions to anti-spacetime



Weyl fermions, black hole and Hawking radiation

Schwarzschild metric

$$ds^2 = -\,dt^2\,(c^2 - v^2) + dr^2/(c^2 - v^2) + r^2 d\Omega^2$$

↑ g_{00} ↑ g_{rr}

singularity at horizon

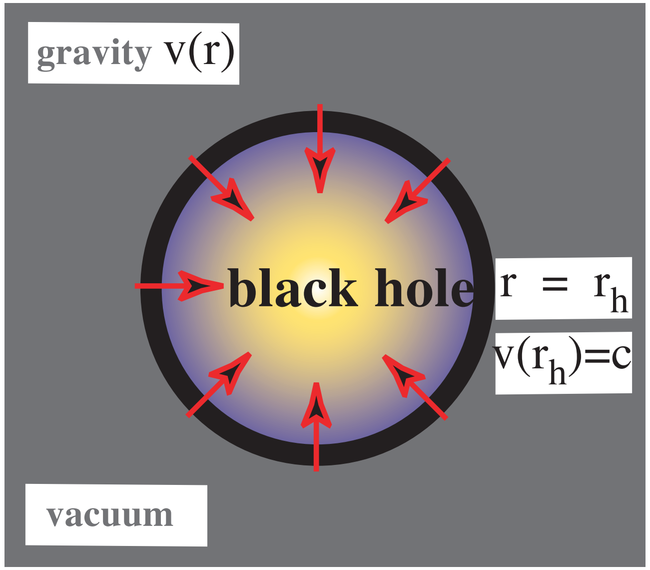
Painleve-Gulstrand metric

$$ds^2 = -\,dt^2\,c^2 + (dr - v\,dt)^2 + dr^2 + r^2 d\Omega^2$$

↑ Doppler shift

no singularity at horizon

$$v^2(r) = \frac{2GM}{r} = c^2\,\frac{r_h}{r}$$



horizon at $g_{00} = 0$ (or $v(r_h) = c$)

v(r) looks like velocity of vacuum

$$v^2(r) = \frac{2GM}{r} = c^2\,\frac{r_h}{r}$$

Weyl fermions in Painleve-Gulstrand spacetime

Weyl fermions in the black hole environment

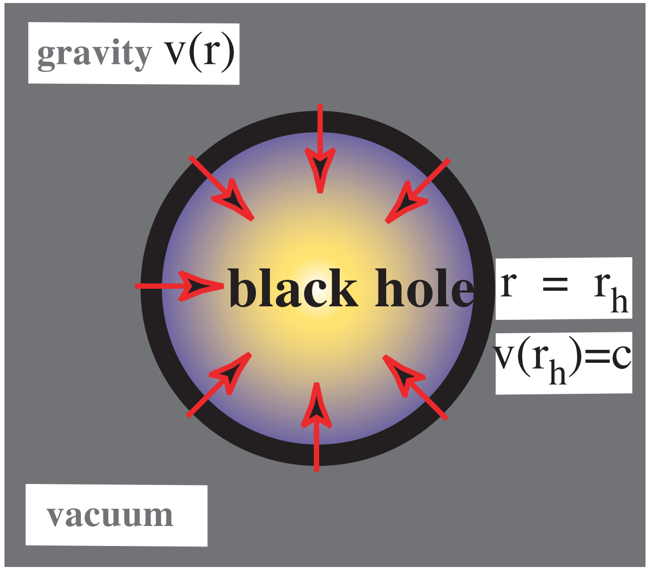
Painleve-Gulstrand metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + (d\mathbf{r} - \mathbf{v} dt)^2$$

Weyl fermions in Painleve-Gulstrand spacetime

$$H = \pm c\boldsymbol{\sigma} \cdot \mathbf{p} - p_r v$$

Doppler shift

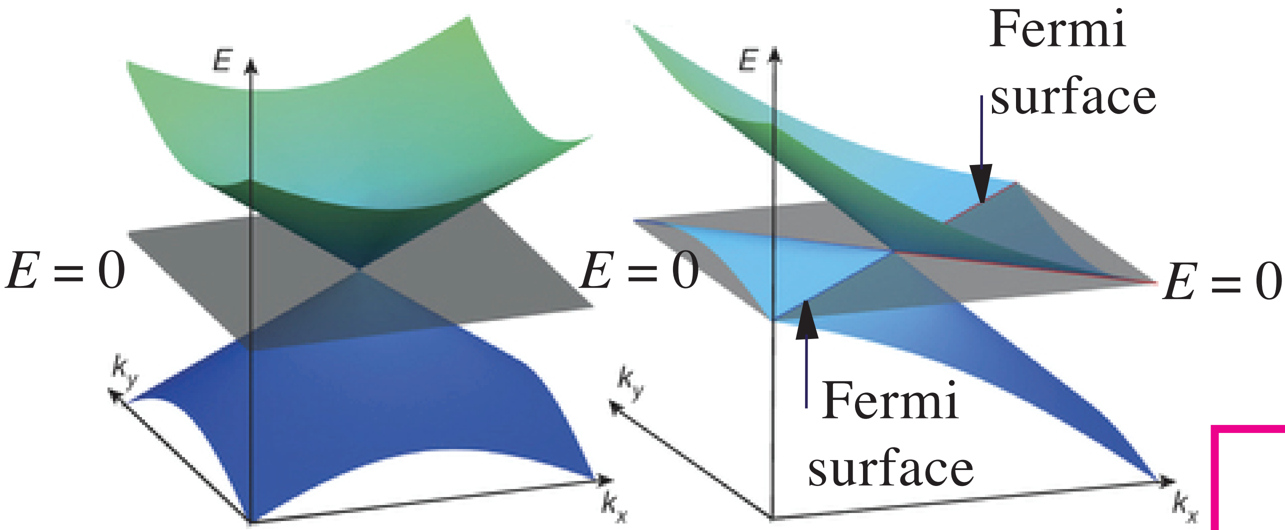


horizon at $g_{00} = 0$ (or $v(r_h) = c$)

$$\mathbf{v}(\mathbf{r}) = -\hat{\mathbf{r}} c \sqrt{\frac{r_h}{r}}$$

at $r > r_h$ $v(r) > c$ type II Weyl point is formed:
two Fermi surfaces connected by Weyl point

type-II Weyl fermions behind horizon

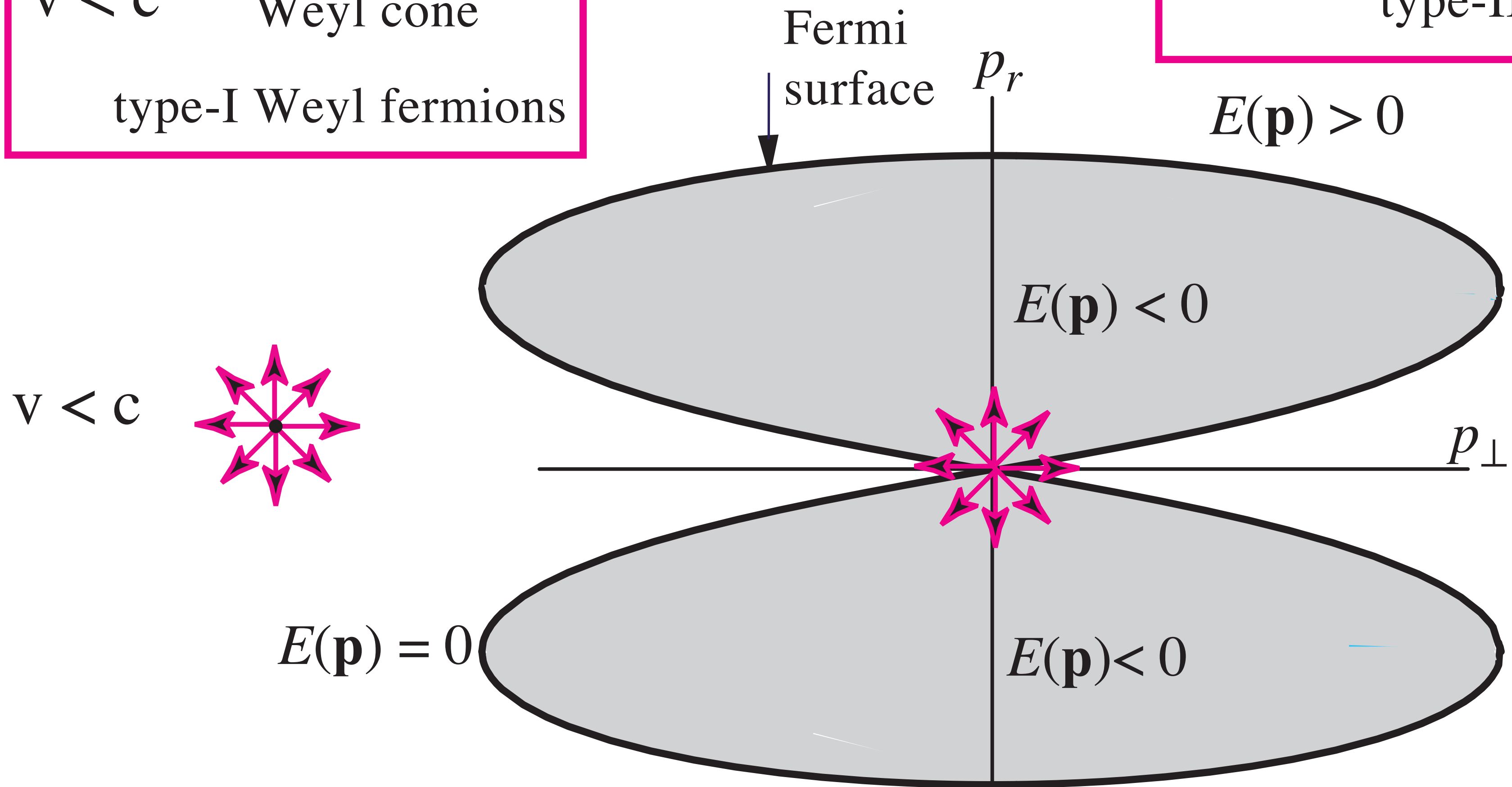


fermionic degrees of freedom inside black hole horizon

$$H = \pm c\boldsymbol{\sigma}\cdot\mathbf{p} - p_r v$$

v > c — overtilted Weyl cone — type-II Weyl fermions

v < c — Weyl cone — type-I Weyl fermions



Fermi surfaces inside horizon connected by Weyl point

(Huhtala, GV 2002)

black hole horizon at interface between type-I and type-II Weyl materials

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + (d\mathbf{r} - \mathbf{v}dt)^2$$

$$\mathbf{v}(\mathbf{r}) = -\hat{\mathbf{r}}c\sqrt{\frac{r_h}{r}}$$

overtilted light cone

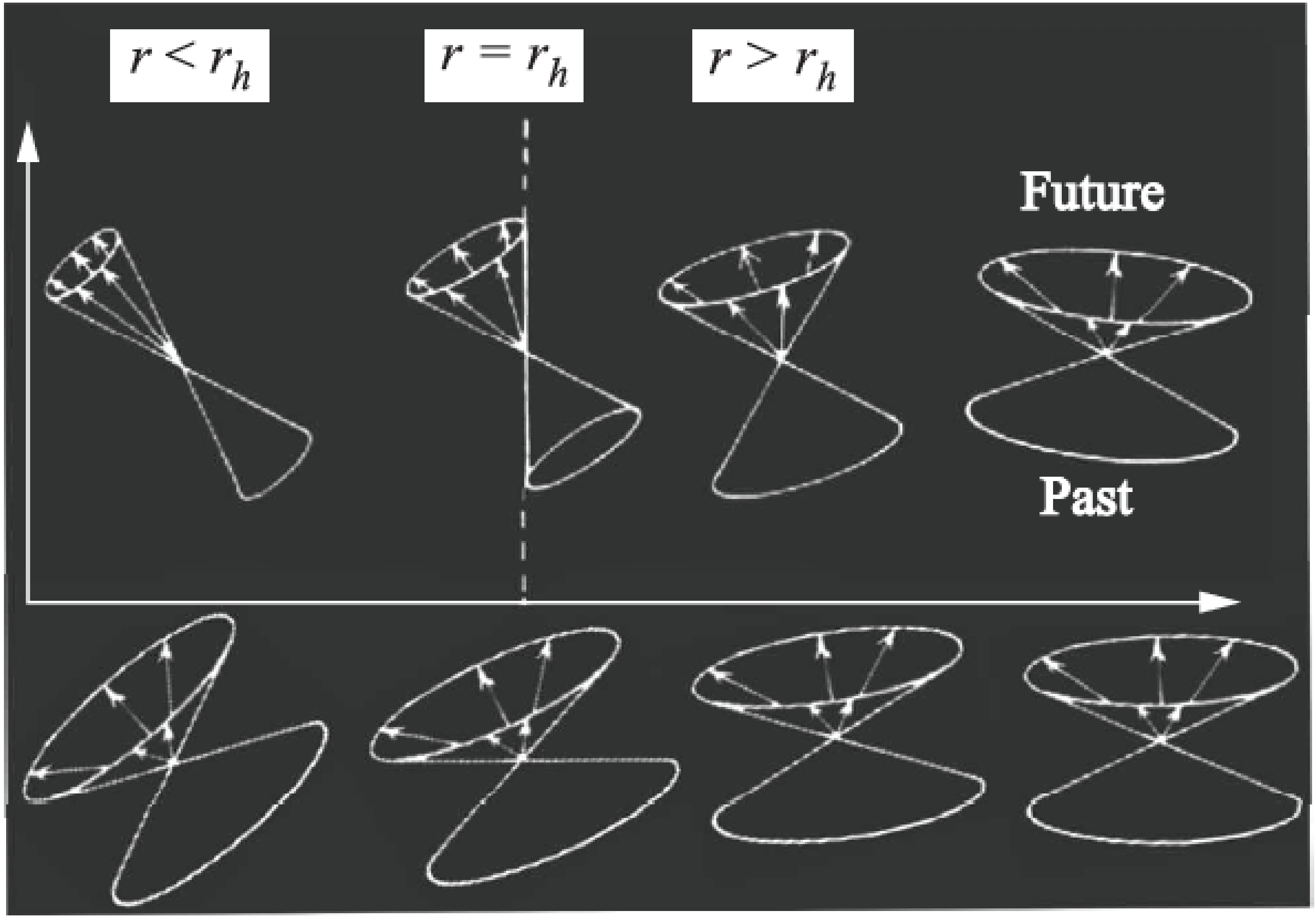
v > c

horizon

light cone

v < c

real space topological Lifshits transition



overtilted Weyl cone

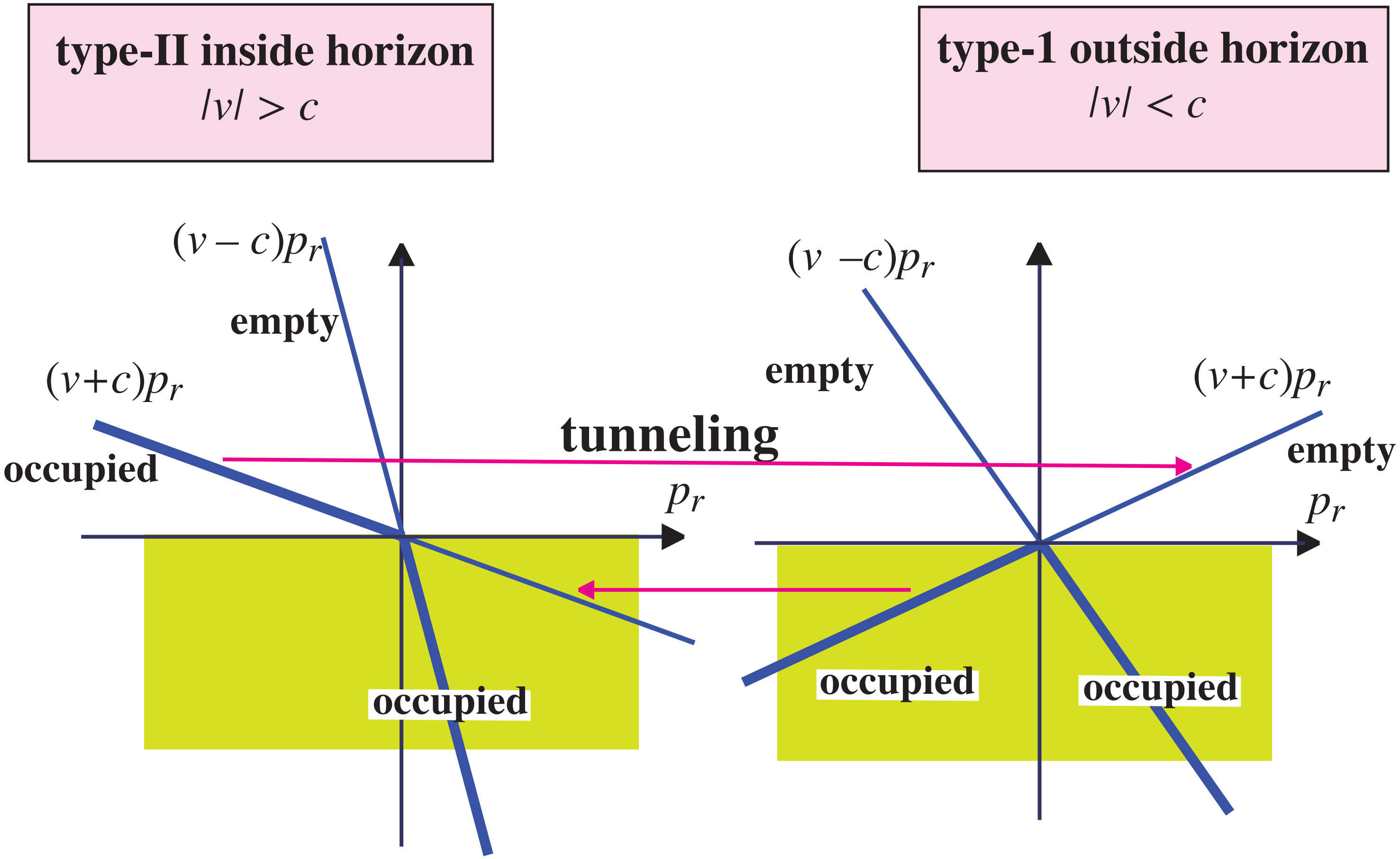
v > c

$$H = \pm c\boldsymbol{\sigma} \cdot \mathbf{p} - p_r v$$

Weyl cone

v < c

Hawking radiation as tunneling

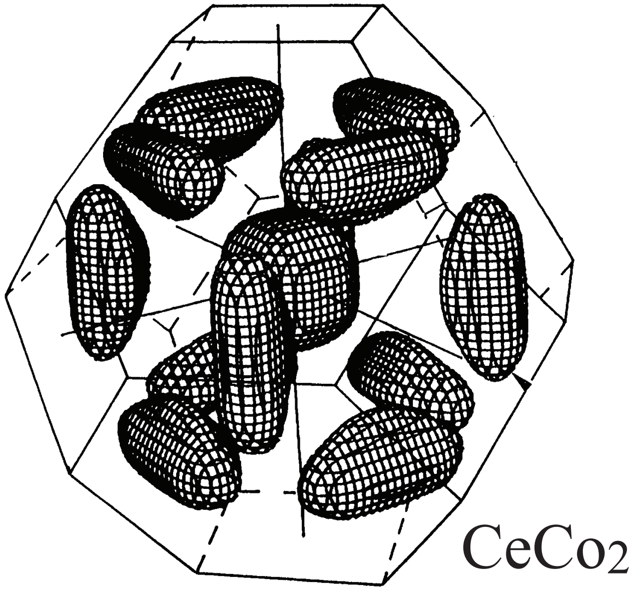


$$T_{\mathrm{H}} = \frac{\hbar}{2\pi}\left(\frac{dv}{dr}\right)_{r=r_h}$$

from:
Gor'kov equations

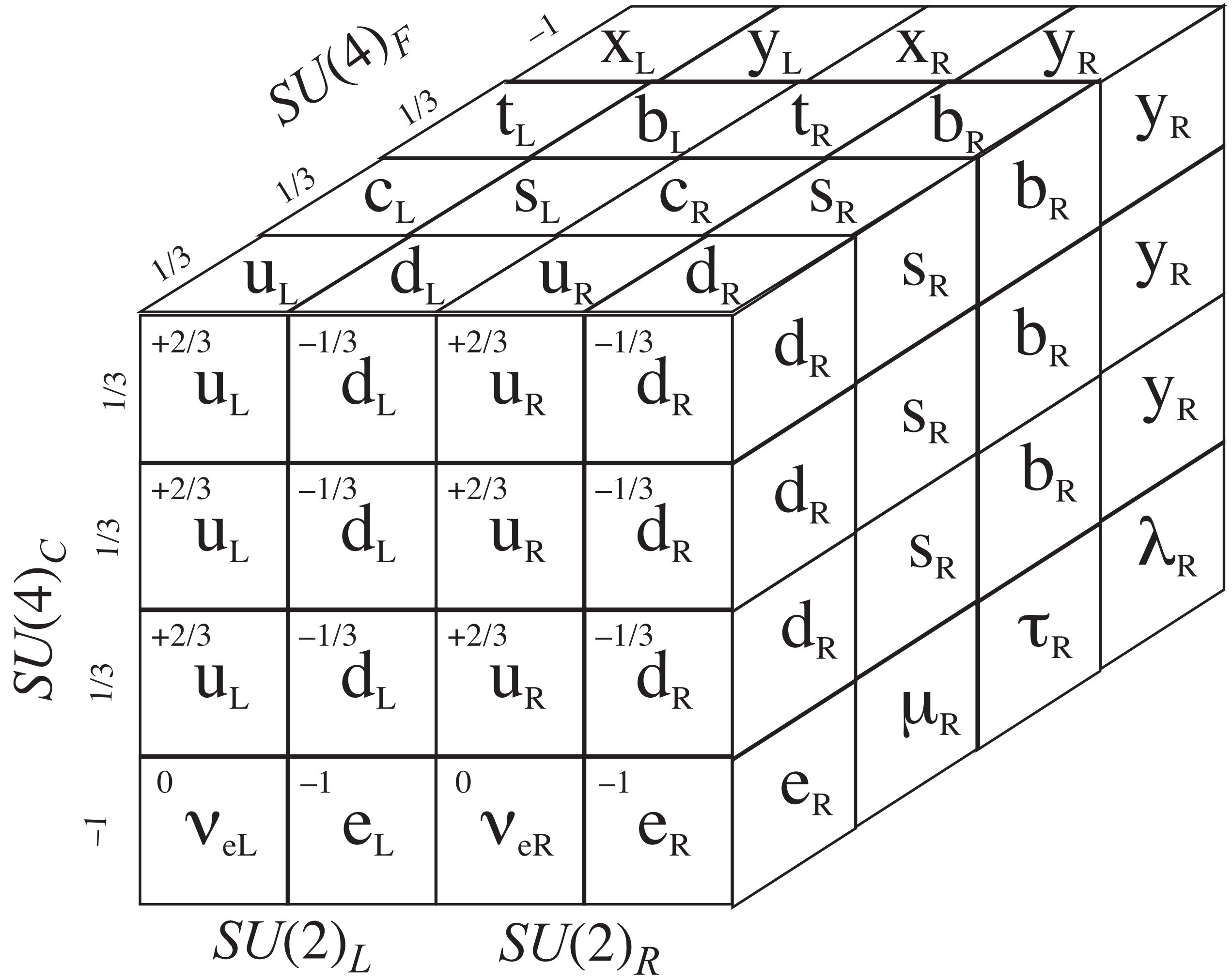
to:

Pati-Salam Model of particle physics
with 4 generations



Agterberg, Barzykin, Gor'kov
(1999)

3 x 4 x 2 + 8 = 32 Dirac fermions



16 x 4 = 64 Weyl fermions

↓

8 x 4 = 32 Dirac fermions

magic 2^N rule

$N = 1$ $= 2$ components of Majorana fermion

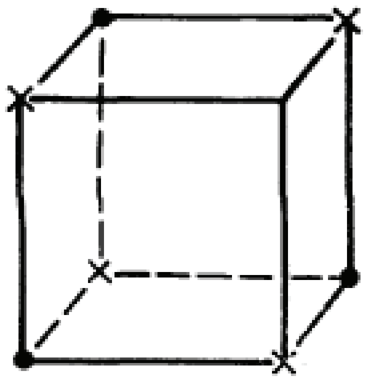
$N = 2$ $= 4$ components of chiral Weyl fermion

$N = 3$ $= 8$ components of Dirac fermion

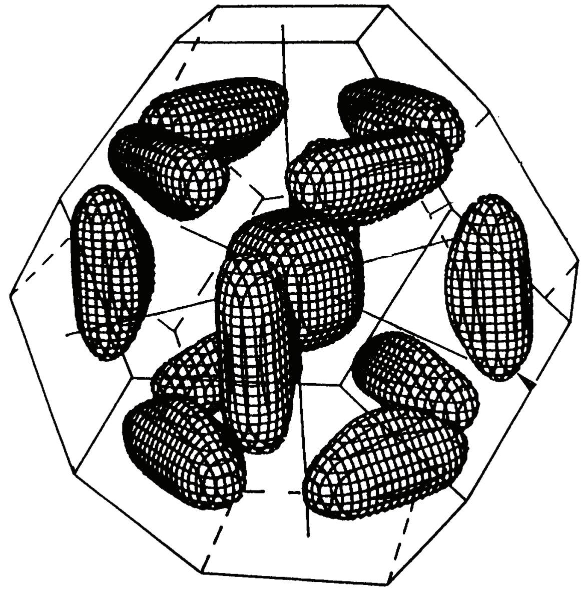
$N = 6$ $= 64$ components of 16 fermions of one generation

$N = 8$ $= 256$ components of fermions of 4 generations

$N = 5$: 8 Weyl nodes in α–phase of cubic superconductor (GV & Gor'kov, 1985)



$N = 6$: 16 Weyl nodes in 4D graphene (Creutz, 2008)



$N = 8$: 64 Weyl nodes in $CeCo_2$ (Agterberg, Barzykin, Gor'kov 1999)