Debye mechanism of microwave absorption in superconductors

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Characteristic relaxation times in metals

Momentum relaxation time τ_{el} is controlled by elastic scattering of electrons from impurities

Inelastic relaxation time τ_{in} is due to electron-electron and electron-phonon scattering

At low temperatures the inelastic relaxation time is much longer than the elastic one

$$\tau_{in} \gg \tau_{el}$$

Microwave absorption in metals

$$\boldsymbol{E}(t) = \boldsymbol{E}_\omega \cos(\omega t)$$

Microwave field penetrates into the metal to skin depth

At small frequencies $\omega \ll \tau_{el}^{-1}$ **microwave absorption is controlled by the *dc* Drude conductivity**

The long inelastic relaxation time plays no role

Microwave absorption in superconductors

For $\omega \ll \Delta \ll T_c$

$$\mathbf{j} = \frac{e}{m} N_{\mathrm{s}}\, \boldsymbol{p}_{\mathrm{s}} + \sigma \mathbf{E}$$

$$\boldsymbol{p}_s = \frac{\hbar}{2} \boldsymbol{\nabla} \chi - \frac{e}{c} \boldsymbol{A}$$

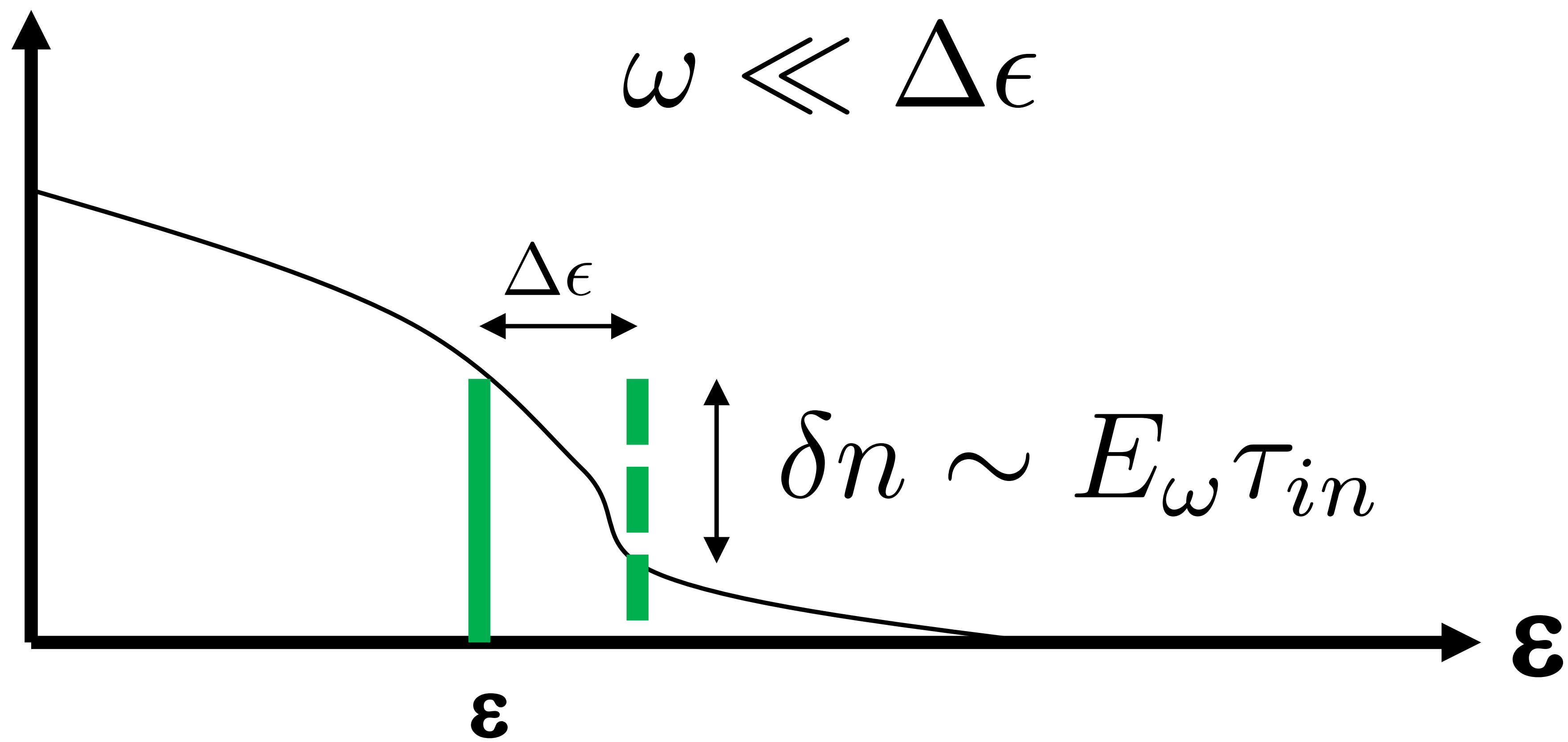
Near the critical temperature the quasiparticle relaxation times are of the same order as in the normal state. As a result the conductivity is of the order of the Drude value

$$\sigma = \sigma_{\mathrm{D}} \ln \frac{T_c}{\omega}$$

Larkin, Ovchinnikov;
Aronov, *et al,* Adv. in Phys. (1981)

The long inelastic relaxation time plays no role

Debye absorption mechanism



Conductivity from entropy production: $T\dot{S} = \frac{\sigma_{DB}}{2} E_\omega^2$

Microwave absorption coefficient is proportional to the inelastic relaxation time

Debye mechanism of microwave absorption in superconductors

The quasiparticle density of states depends on p_s

$$\nu(\epsilon, p_s) \qquad\qquad \boldsymbol{p}_s = \frac{\hbar}{2}\boldsymbol{\nabla}\chi - \frac{e}{c}\boldsymbol{A}$$

In the presence of microwave field $\nu(\epsilon, p_s)$ **depends on time:**

$$\dot{\boldsymbol{p}}_s(t) = e\mathbf{E}(t)$$

The motion of quasiparticle levels in energy space creates non-equilibrium quasiparticle distribution. Its relaxation increases entropy and contributes to microwave absorption.

Since the density of states is even in p_s **the Debye contribution exists only in the nonlinear regime or in the presence of a *dc* supercurrent.**

Quasiparticle dynamics in the presence of uniform microwave field

$\tau_{in} \gg \tau_{el}$ **At low frequencies the quasiparticle distribution function depends only on the energy:** $n(\epsilon, t)$

Two continuity equations:

Number of quasiparticle states is conserved

$$\partial_t \nu(\epsilon, p_s) + \partial_\epsilon \left(v_\nu(\epsilon) \nu(\epsilon, p_s) \right) = 0$$

$v_\nu(\epsilon) = e\boldsymbol{E} \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_{\mathrm{s}})$ **velocity of quasiparticle levels in energy space**

$$\boldsymbol{V}(\epsilon, \boldsymbol{p}_{\mathrm{s}}) = -\frac{1}{\nu(\epsilon, p_{\mathrm{s}})} \int_0^\epsilon d\tilde{\epsilon} \frac{\partial \nu(\tilde{\epsilon}, p_{\mathrm{s}})}{\partial \boldsymbol{p}_{\mathrm{s}}}$$

In the absence of energy relaxation the quasiparticle distribution function follows the energy levels

$$\partial_t(\nu n) + \partial_\epsilon(v_\nu \nu n) = 0$$

Kinetic equation in the presence of level motion

$$\partial_t n(\epsilon, t) + e\boldsymbol{E}(t) \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_{\mathrm{s}})\, \partial_\epsilon n(\epsilon, t) = I\{n\}$$

$$\dot{\boldsymbol{p}}_s(t) = e\mathbf{E}(t) \qquad \boldsymbol{V}(\epsilon, \boldsymbol{p}_{\mathrm{s}}) = -\frac{1}{\nu(\epsilon, p_{\mathrm{s}})} \int_0^\epsilon d\tilde{\epsilon} \frac{\partial \nu(\tilde{\epsilon}, p_{\mathrm{s}})}{\partial \boldsymbol{p}_{\mathrm{s}}}$$

Absorbed microwave power

$$W = \int_0^\infty d\epsilon \langle \nu(\epsilon, p_s(t)) n(\epsilon, t) e\boldsymbol{E}(t) \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_s(t)) \rangle = \sigma_{DB} \frac{E_\omega^2}{2}$$

$\langle \ldots \rangle$ **— averaging over the oscillation period**

Debye contribution to the conductivity

Linear absorption in the presence of *dc* supercurrent

$$\boldsymbol{p}_{\mathrm{s}}(t) = \bar{\boldsymbol{p}}_{\mathrm{s}} + \delta \boldsymbol{p}_{\mathrm{s}}(t) \qquad \delta \dot{\boldsymbol{p}}_s(t) = e\mathbf{E}(t)$$

Near T_c the relevant to the Debye mechanism energies are much smaller than T, while the energy transfer in the relaxation processes is of order T.
Therefore the inelastic collision integral may be described by the relaxation time approximation:

$$I\{n\} = -\frac{\delta n(\epsilon, t)}{\tau_{in}} = \frac{n_F(\epsilon) - n(\epsilon, t)}{\tau_{in}}$$

Solution of the kinetic equation:

$$\delta n_\omega(\epsilon) = -\frac{e\boldsymbol{E}_\omega \cdot \boldsymbol{V}(\epsilon, \bar{\boldsymbol{p}}_s)}{-i\omega + \tau_{in}^{-1}} \frac{dn_F(\epsilon)}{d\epsilon}$$

$$\Downarrow$$

$$W = \int_0^\infty d\epsilon \langle \nu(\epsilon, p_s(t)) n(\epsilon, t) e\boldsymbol{E}(t) \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_s(t)) \rangle = \sigma_{DB} \frac{E_\omega^2}{2}$$

Debye contribution to the longitudinal conductivity

Near Tc the ratio of the Debye contribution to the Drude result is

$$\frac{\sigma_{DB}}{\sigma_D} = \frac{3\tau_{in}}{4\tau_{el}} \frac{1}{\left[1 + (\omega\tau_{in})^2\right]} \int_0^\infty \frac{d\epsilon}{T} \frac{\nu(\epsilon, \bar{p}_s) V^2(\epsilon, \bar{p}_s)}{\nu_n v_F^2}$$

$$\boldsymbol{V}(\epsilon, \boldsymbol{p}_\mathrm{s}) = -\frac{1}{\nu(\epsilon, p_\mathrm{s})} \int_0^\epsilon d\tilde{\epsilon} \frac{\partial \nu(\tilde{\epsilon}, p_\mathrm{s})}{\partial \boldsymbol{p}_\mathrm{s}}$$

- **The Debye contribution to the conductivity is strongly anisotropic (depends on the angle between the microwave field and supercurrent).**
- **It can be expressed in terms of the dependence of the density of states on** p_s

Dependence of the density of states on superfluid current

Singularity in the BCS density of states is broadened at $p_s \neq 0$

The width $\delta\epsilon$ and shape of the broadening depend on p_s and strength of disorder

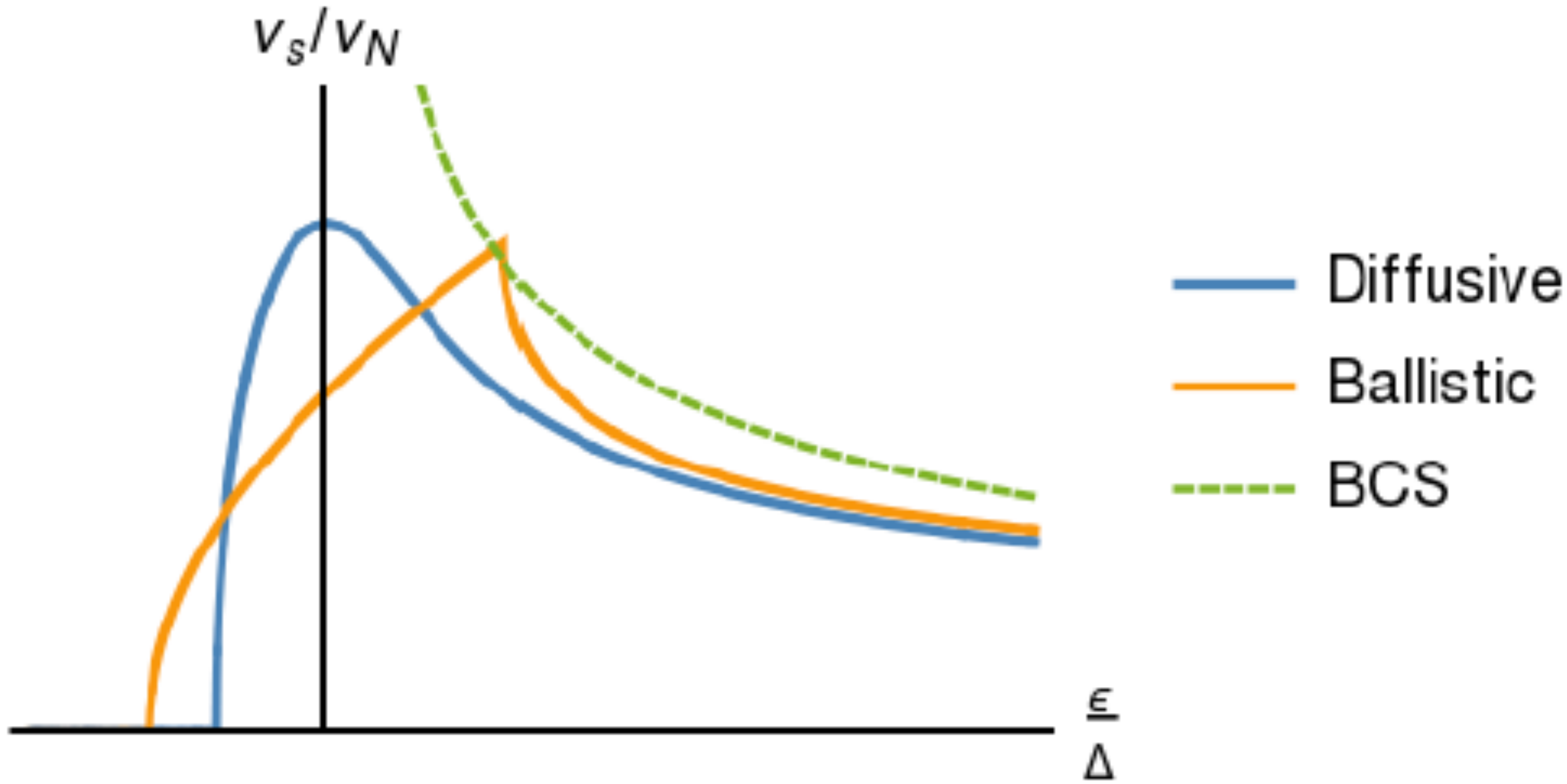


Figure 1: Schematic plots of $\nu(\epsilon, p_\mathrm{s})$ at: $p_\mathrm{s} = 0$ - dashed green line, in the diffusive regime $\Delta v_F p_\mathrm{s} \tau_\mathrm{el}^2 \ll 1$ - blue line, and in the ballistic regime $\Delta v_F p_\mathrm{s} \tau_\mathrm{el}^2 \gg 1$ – orange line.

Simplest case - clean limit

$$v_{\mathrm{F}}\bar{p}_{\mathrm{s}}\tau_{\mathrm{el}}^2\Delta \gg 1$$

Quasiparticle spectrum: $\epsilon(\boldsymbol{k}) = \sqrt{\xi_{\boldsymbol{k}}^2 + |\Delta|^2} + \boldsymbol{v}_{\boldsymbol{k}} \cdot \bar{\boldsymbol{p}}_{\mathrm{s}}$

Broadening width: $\delta\epsilon \sim v_F \bar{p}_{\mathrm{s}}$

Density of states

$$\frac{\nu(\epsilon, p_s)}{\nu_n} = \sqrt{\frac{\Delta}{2v_{\mathrm{F}}p_{\mathrm{s}}}}\left[\theta(z+1)\sqrt{z+1} - \theta(z-1)\sqrt{z-1}\right]$$

$$z = (\epsilon - \Delta)/v_F p_{\mathrm{s}}$$

Conductivity in the ballistic regime

$$\frac{\sigma_{\mathrm{DB}}}{\sigma_{\mathrm{D}}} = \frac{8}{45}\frac{\tau_{\mathrm{in}}}{\tau_{\mathrm{el}}\left[1+(\omega\tau_{\mathrm{in}})^2\right]}\frac{\Delta}{T}\sqrt{\frac{v_{\mathrm{F}}\bar{p}_{\mathrm{s}}}{\Delta}}$$

- **Debye contribution to the conductivity is proportional to the inelastic relaxation time at low frequencies**
- **It is strongly anisotropic**
- **Its dependence on the supercurrent density is non-analytic**

Range of applicability of ballistic result

Quasiparticle spectrum: $\epsilon(\boldsymbol{k}) = \sqrt{\xi_{\boldsymbol{k}}^2 + |\Delta|^2} + \boldsymbol{v_k} \cdot \bar{\boldsymbol{p}}_{\mathrm{s}}$

Elastic quasiparticle relaxation rate: $\dfrac{1}{\tau_{el}(\epsilon)} = \dfrac{1}{\tau_{el}} \sqrt{\dfrac{\epsilon - \Delta}{\Delta}}$

Ballistic regime: $v_F \bar{p}_{\mathrm{s}} \tau_{el}(\epsilon) \gg 1$

$$v_{\mathrm{F}} \bar{p}_{\mathrm{s}} \tau_{\mathrm{el}}^2 \Delta \gg 1$$

General case: Gor'kov equations

Density of states: $$\frac{\nu(\epsilon, p_{\rm s})}{\nu_n} = -\frac{2}{\pi}\,{\rm Im}\, g(\epsilon)$$

$$g(\epsilon) = \left\langle \frac{\tilde\epsilon - \boldsymbol{v}\cdot\boldsymbol{p}_{\rm s}}{\sqrt{(\tilde\epsilon - \boldsymbol{v}\cdot\boldsymbol{p}_{\rm s})^2 - |\tilde\Delta|^2}} \right\rangle$$

$\langle \ldots \rangle$ - **Fermi surface averaging**

$$\tilde\epsilon \quad = \quad \epsilon + \frac{i}{2\tau_{el}} \left\langle \frac{\tilde\epsilon - \boldsymbol{v}\cdot\boldsymbol{p}_s}{\sqrt{(\tilde\epsilon - \boldsymbol{v}\cdot\boldsymbol{p}_{\rm s})^2 - |\tilde\Delta|^2}} \right\rangle,$$

$$\tilde\Delta \quad = \quad \Delta + \frac{i}{2\tau_{el}} \left\langle \frac{\tilde\Delta}{\sqrt{(\tilde\epsilon - \boldsymbol{v}\cdot\boldsymbol{p}_s)^2 - |\tilde\Delta|^2}} \right\rangle,$$

Energies near the gap

Density of states: $\nu(\epsilon, p_s) = \frac{\nu_n}{\sqrt{2}} \Im x^{-1}$

Quintic equation: $x\left(x^2 + w\right)\left(2x + \frac{\gamma}{\sqrt{2}}\right)^2 = -\zeta^2\left(x + \frac{\gamma}{3\sqrt{2}}\right)$

Here: $w = (\epsilon - \Delta)/\Delta, \ \gamma = 1/(\tau_{\rm el}\Delta), \ \zeta = v_{\rm F} p_{\rm s}/\Delta$

energy

disorder strength

supercurrent density

Limiting regimes

High supercurrent: $\zeta/\gamma^2 = v_F \bar{p}_{\rm s} \tau_{\rm el}^2 \Delta \gg 1$ **- ballistic regime** $x \gg \gamma$

Low supercurrent: $\zeta/\gamma^2 \ll 1$ **- diffusive regime** $x \ll \gamma$

Cubic equation: $x\left(x^2 + w\right) + \frac{\sqrt{2}\zeta^2}{3\gamma} = 0$ **(also follows from Usadel equation)**

Diffusive regime: $v_F \bar{p}_{\mathrm{s}} \tau_{\mathrm{el}}^2 \Delta \ll 1$.

Dirty superconductors and clean superconductors at weak supercurrents

Broadening width of the BCS peak: $\delta\epsilon \sim \left(\Delta D^2 \bar{p}_{\mathrm{s}}^4\right)^{1/3}$

Debye contribution to the conductivity

$$\frac{\sigma_{\mathrm{DB}}}{\sigma_{\mathrm{D}}} = I_{\mathrm{d}} \frac{\tau_{\mathrm{in}}}{\tau_{\mathrm{el}}} \frac{\Delta}{T} \frac{\tau_{\mathrm{el}} \left(\Delta D^2 \bar{p}_{\mathrm{s}}^4\right)^{1/3}}{1 + \left(\omega \tau_{\mathrm{in}}\right)^2}$$

$I_{\mathrm{d}} \approx 0.055$

Yu. N. Ovchinnikov, A. R. Isaakyan, JETP **74**, 178 (1978)

Nonlinear regime

The Debye contribution to the nonlinear conductivity exists in the absence of *dc* supercurrent. The nonlinear threshold is anomalously low.

$$\dot{\boldsymbol{p}}_s(t) = e\mathbf{E}(t)$$

Density of states - nonlinear function of time

$$\partial_t n(\epsilon, t) + e\boldsymbol{E}(t) \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_{\mathrm{s}})\, \partial_\epsilon n(\epsilon, t) = I\{n\}$$

For $\delta\epsilon \ll T$ **the inelastic collision integral may be linearized**

Solution in Langrangian variables

Qualitatively: $\bar{p}_{\mathrm{s}} \to \dfrac{eE_\omega}{\omega}$ **in linear results**

Nonlinear conductivity

Ballistic regime: $eE_\omega v_\mathrm{F}\Delta\tau_\mathrm{el}^2 \gg \omega$

$$\frac{\sigma_\mathrm{DB}^\mathrm{nl}}{\sigma_\mathrm{D}} = \frac{\tau_\mathrm{in}}{\tau_\mathrm{el}}\frac{\Delta}{T}\sqrt{\frac{v_\mathrm{F}eE_\omega}{\omega\Delta}}\,F_\mathrm{b}(\omega\tau_\mathrm{in})$$

Diffusive regime: $eE_\omega v_\mathrm{F}\Delta\tau_\mathrm{el}^2 \ll \omega$

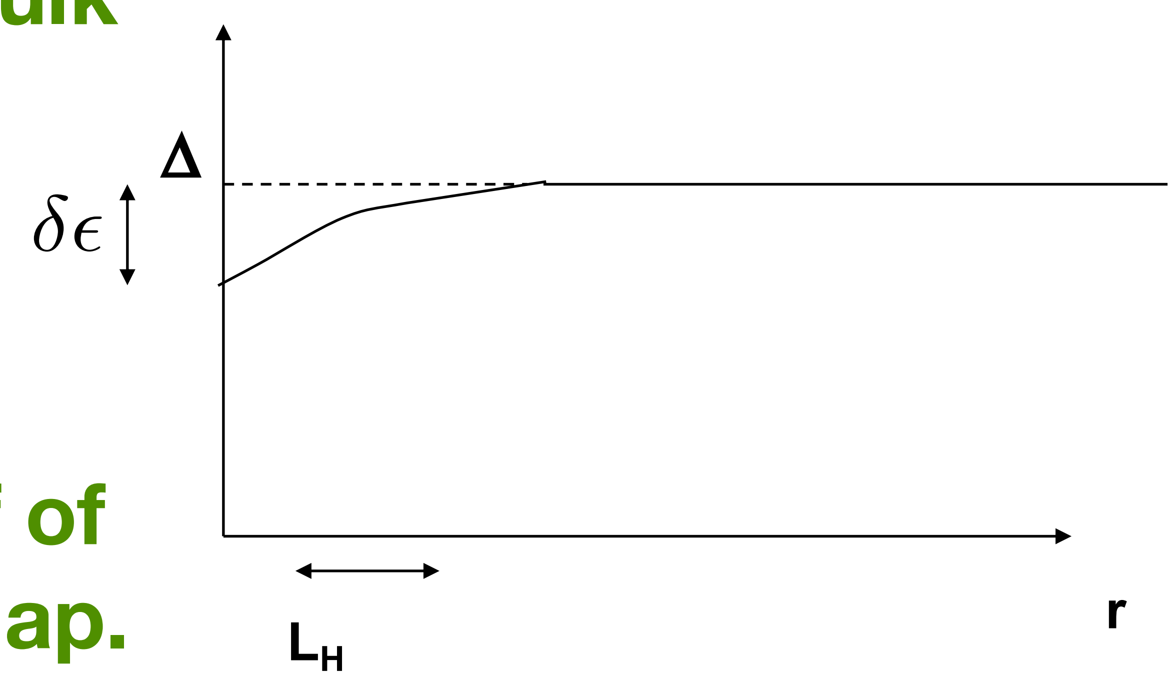
$$\frac{\sigma_\mathrm{DB}^\mathrm{nl}}{\sigma_\mathrm{D}} = \frac{\tau_\mathrm{in}}{\tau_\mathrm{el}}\frac{\Delta}{T}\frac{\Delta^{1/3}D^{5/3}|eE|^{4/3}}{v_F^2\omega^{4/3}}\,F_\mathrm{d}(\omega\tau_\mathrm{in})$$

Here: $F_\mathrm{b}(0)\approx 0.108, \quad F_\mathrm{d}(0) = 0.109$ $\qquad F_{b,d}(x) \propto \frac{1}{x^2}, \;\text{at}\;\; x \gg 1$

Caveats

- Nonanalytic dependence of the Debye contribution to the conductivity on $\bar{p}_{\mathrm{s}}$ and E_ω arises from the BCS singularity in the density of states. In real situations the singularity is broadened by the gap anisotropy and inelastic scattering. Thus at very small $\bar{p}_{\mathrm{s}}$, E_ω this dependence is analytic.

- Microwave field penetrates into bulk superconductors to distances of order of the London length L_H. Quasiparticles that diffuse out of this layer do contribute to Debye absorption. However, roughly half of the relevant states lie below the gap. Thus the microwave absorption coefficient will be of the same order as in a film of thickness L_H.



Summary

- **Debye contribution to conductivity is proportional to the inelastic relaxation time and may be much larger than the conventional contribution. This may enable determination of the inelastic relaxation time from microwave absorption measurements.**

- **Debye contribution to the linear conductivity exists only in the presence of supercurrent and exhibits anisotropy with respect to relative orientation between E_ω and $\bar{p}_{\rm s}$.**

- **In the nonlinear regime it is present even in the absence of supercurrent.**