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Normal and superfluid He-3 in anisotrpic aerogel: phase diagram and spin current

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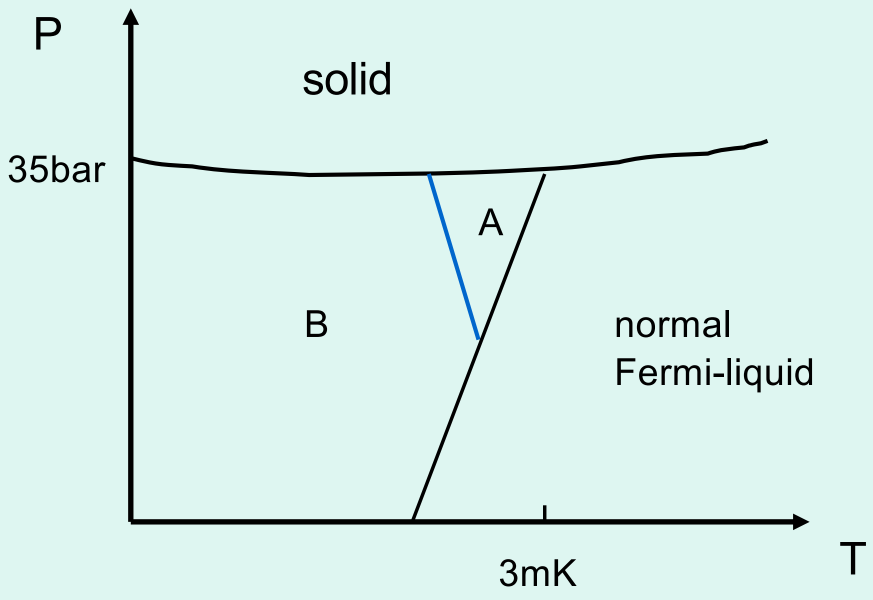
Outline

- Phase diagram of superfluid He-3
  in anisotropic aerogel
- Phase diagram dependence from
  aerogel density and degree of anisotropy
- Role of exchange scattering
- Experimental phase diagrams
- Microscopic theory
- Spin current
- Conclusion

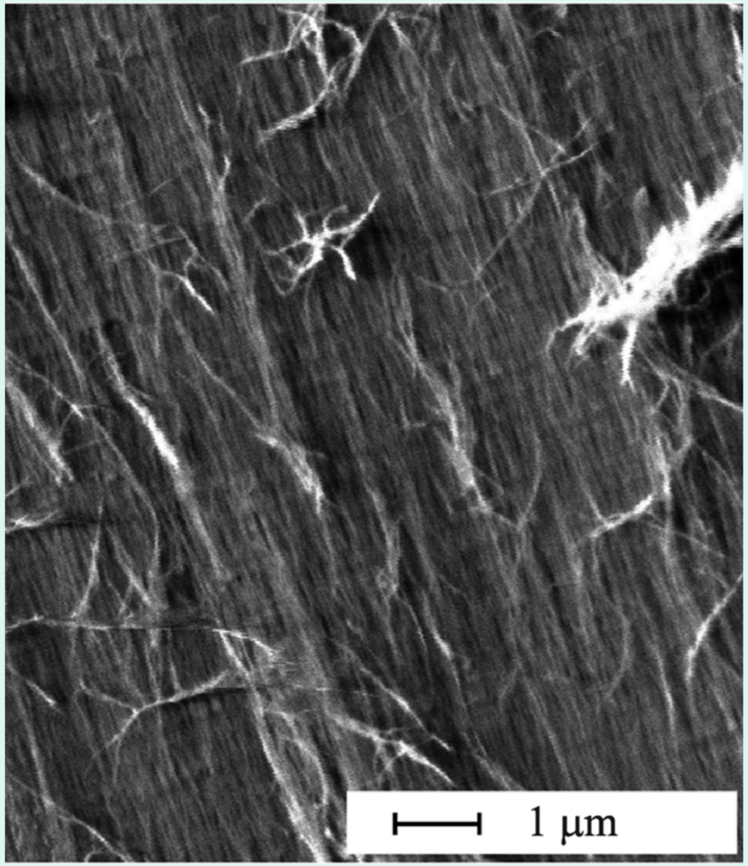
Superfluid He-3



$$\Psi_A \propto (k_x + ik_y)(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$\Psi_B \propto (-k_x + ik_y)(|\uparrow\uparrow\rangle + k_z(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) + (k_x + ik_y)|\downarrow\downarrow\rangle$$

$$\Psi_{pol} \propto k_z(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$



Polar state

Nematically ordered aerogel Nafen – 90

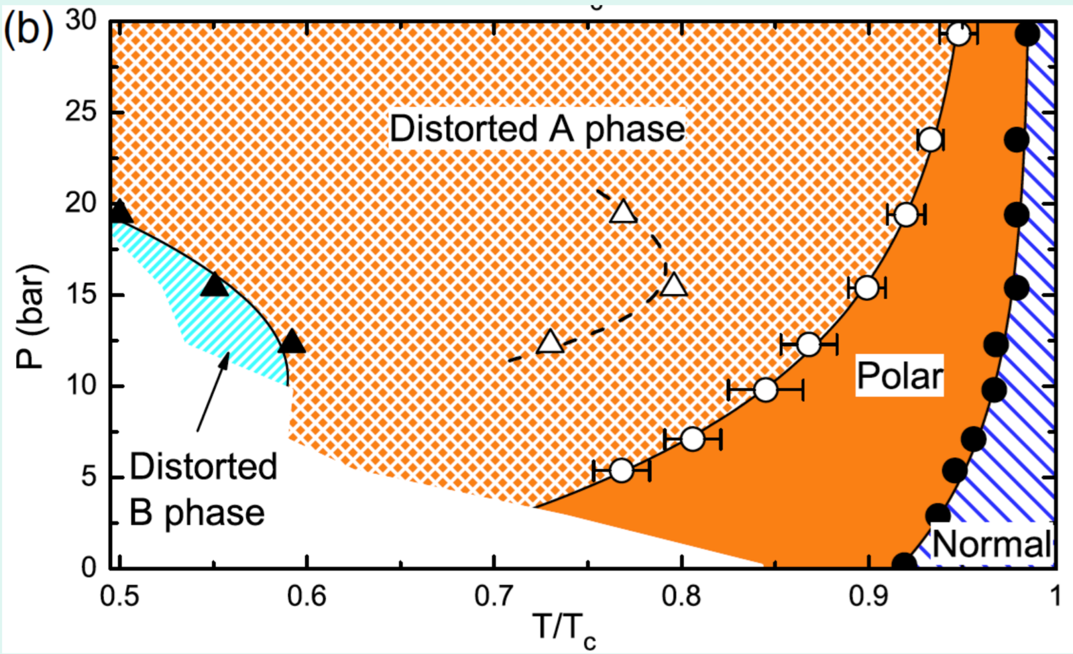
Al_2O_3 strands almost parallel at distance 3 – 5 mm

mean diameter 8 nm, mean distance 50 nm

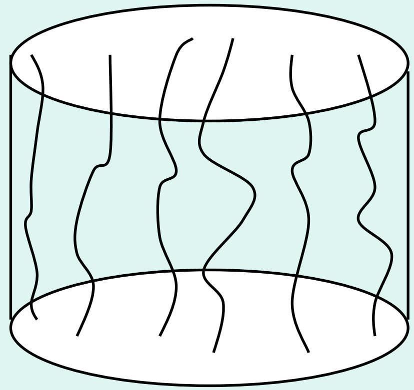
Asadchikov et al, JETP Lett. 2015

Theory - Aoyama, Ikeda, PRB 2006
Experiment - Dmitriev, Senin, Soldatov, Yudin, PRL 2015

$$\Psi_{pol} \propto k_z(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$



He-3 superfluid critical temperature in anisotropic aerogel



Anisotropy lifts the degeneracy between superfluid phases

$$F^{(2)} = F_i^{(2)} + F_a^{(2)} = \alpha_0 (T - T_c)|\Delta|^2 + \eta_{ij} A_{\alpha i} A^{\star}_{\alpha j}$$

Media uniaxial anisotropy with anisotropy axis along $\hat{z}$ is given by the traceless tensor

$$\eta_{ij} = \eta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\eta > 0$$

(iv) Polar phase $\quad A_{\alpha i} = \Delta V_\alpha \hat{z}_i e^{i\phi} \quad F_a^{(2)} = -2\eta|\Delta|^2 \quad T_c^{polar} = T_c + 2\eta/\alpha_0$

Axipolar phase formation in uniaxially anisotropic aerogel

$$F_{cond} = \alpha A^{\star}_{\alpha i} A_{\alpha i} + \eta_{ij} A_{\alpha i} A^{\star}_{\alpha j}$$
$$+\beta_1 |A_{\alpha i} A_{\alpha i}|^2 + \beta_2 A^{\star}_{\alpha i} A_{\alpha j} A^{\star}_{\beta i} A_{\beta j} + \beta_3 A^{\star}_{\alpha i} A_{\beta i} A^{\star}_{\alpha j} A_{\beta j} + \beta_4 (A^{\star}_{\alpha i} A_{\alpha i})^2 + \beta_5 A^{\star}_{\alpha i} A_{\beta i} A^{\star}_{\beta j} A_{\alpha j}$$

Axipolar phase $A_{\alpha i} = V_\alpha \left[a\hat{z}_i + ib(\hat{x}_i \cos\varphi(\mathbf{r}) + \hat{y}_i \sin\varphi(\mathbf{r}))\right]$

$$F_{cond} = (\alpha - 2\eta) a^2 + (\alpha + \eta) b^2 + \beta_{12} (a^2 - b^2)^2 + \beta_{345} (a^2 + b^2)^2$$

$$\alpha = \alpha_0 (T - T_c)$$

$$\beta_{12} = \beta_1 + \beta_2, \qquad \beta_{345} = \beta_3 + \beta_4 + \beta_5$$

$$\beta_{12345} = \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5$$

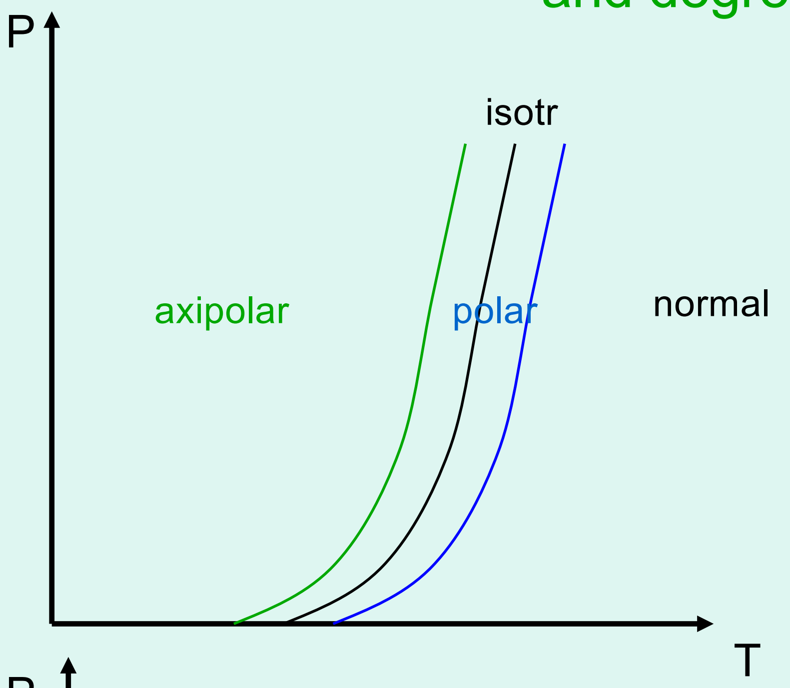
polar $\quad T_{c1} = T_c + 2\frac{\eta}{\alpha_0} \qquad a^2 = a_0^2 = -\frac{\alpha_0 (T - T_{c1})}{2\beta_{12345}}, \quad b = 0.$ $\quad T_{c1} > T_c$

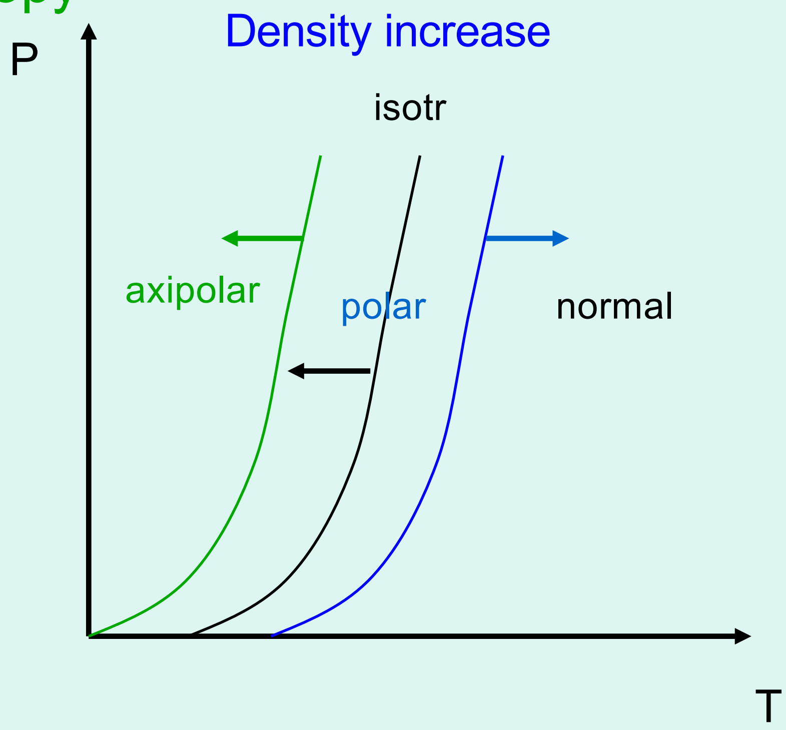
axipolar $\quad T_{c2} = T_c - \frac{\eta}{\alpha_0} \frac{3\beta_{345} - \beta_{12}}{2\beta_{12}} = T_c - \frac{5 - 1.2\delta}{2 + 0.15\delta} \frac{\eta}{\alpha_0}$ $\quad T_{c2} < T_c$

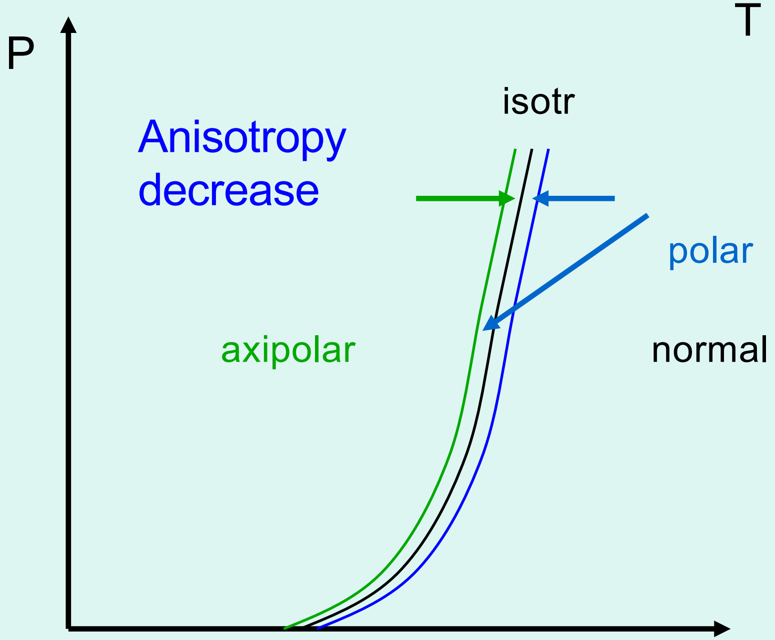
$$a = a_0 + \delta a, \qquad \delta a = -\frac{\beta_{345} - \beta_{12}}{2\beta_{12345}} \frac{b^2}{a_0}, \qquad b^2 = -\frac{\alpha_0 (T - T_{c2})}{4\beta_{345}}$$

VM, J.Low Temp.Phys 2014

Phase diagram dependence from aerogel density and degree of anisotropy







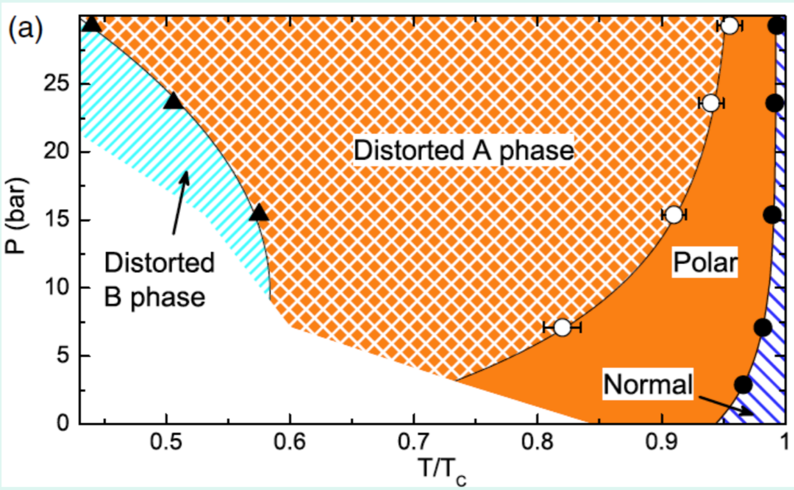
$$T_c^{bulk} - T_c^{isotr} \propto n_{aerogel}$$

$$\eta \propto \delta \times n_{aerogel}$$

$$n_{aerogel} - \ aerogel \ \ density$$

$$\delta - \ degree \ \ of \ \ anisotropy$$

Experimental phase diagrams



$n_a = const$

$\delta \to 0$

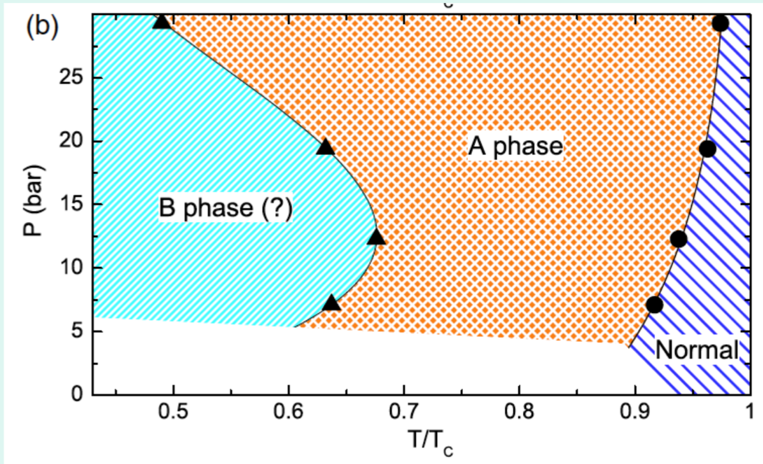
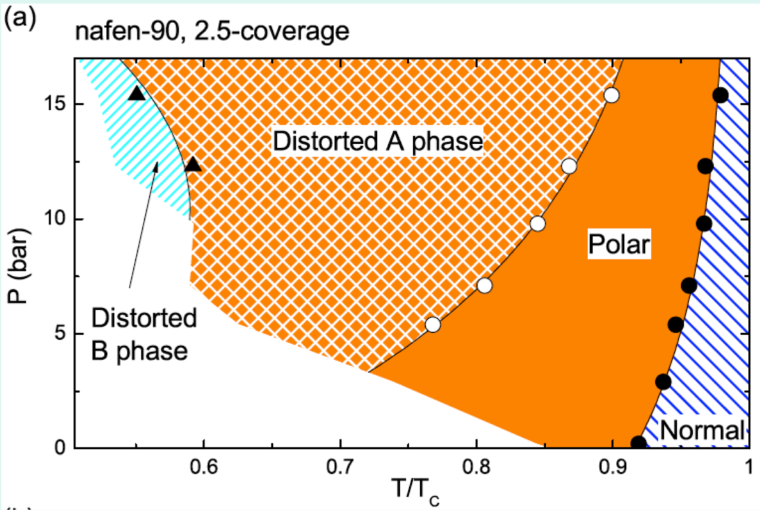


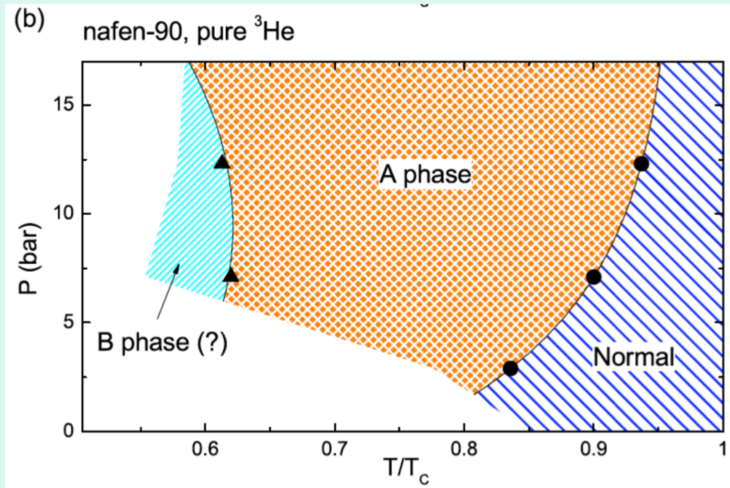
FIG. 3. Phase diagrams of ^{3}He in nafen-72 for 2.5 coverage (a) and for pure ^{3}He (b) obtained on cooling from the normal phase.

$n_a \nearrow$ $\delta = const$



$n_a = const$

$\delta \to 0$



V.V.Dmitriev, A.A.Soldatov, A.N.Yudin, PRL 120,075301 (2018)

Microscopic theory

$$H_{int} = \sum_i \int d^3r \psi^\dagger_\alpha(\mathbf{r}) \left[u(\mathbf{r}-\mathbf{r}_i)\delta_{\alpha\beta} + J(\mathbf{r}-\mathbf{r}_i)\boldsymbol{\sigma}_{\alpha\beta}\mathbf{S} \right] \psi_\beta(\mathbf{r})$$

$$F^{(2)} = \frac{1}{3}\left\{ \frac{1}{g}\delta_{ij} - T\sum_\omega \int \frac{d^3p}{(2\pi)^3}\hat{p}_i \Gamma_j(\mathbf{p},\omega,)G(\mathbf{p},\omega)G(-\mathbf{p},-\omega) \right\} A^\star_{\mu i}A_{\mu j}$$

$$G(\mathbf{p},\omega) = \frac{1}{i\omega - \xi_\mathbf{p} - \Sigma_\mathbf{p}(\omega)}$$

$$\Sigma_\mathbf{p}(\omega) = \int \frac{d^3p'}{(2\pi)^3} U^2_{\mathbf{p}-\mathbf{p}'} G(\mathbf{p}',\omega)$$

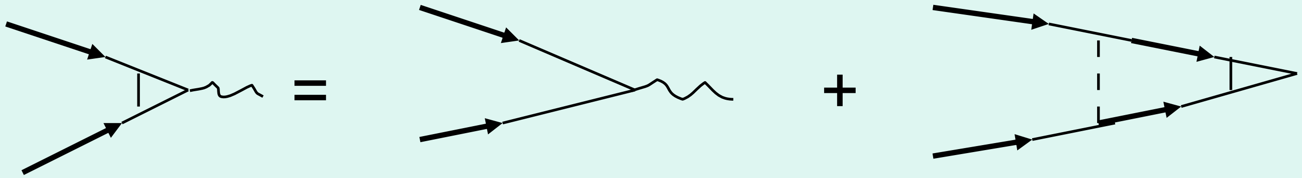
$$U^2_\mathbf{p} = n_i \left[u^2_\mathbf{p} + \langle S_i \sigma^i_{\alpha\gamma} S_k \sigma^k_{\gamma\alpha} \rangle J^2_\mathbf{p} \right] = n_i \left[u^2_\mathbf{p} + \frac{1}{4}S(S+1)J^2_\mathbf{p} \right]$$

$$\Sigma_\mathbf{p}(\omega) = -\frac{i}{2\tau}\left\{ 1 - \delta\left[\hat{p}^2_z - \frac{1}{2}(\hat{p}^2_x + \hat{p}^2_y) \right] \right\} sign\ \omega$$

$$\frac{1}{\tau} = \frac{1}{\tau_p} + \frac{1}{\tau_{ex}}, \qquad \frac{\delta}{\tau} = \frac{\delta_p}{\tau_p} + \frac{\delta_{ex}}{\tau_{ex}}$$

$$\Gamma_j(\mathbf{p},\omega) = \hat{p}_j + n\int \frac{d^3p'}{(2\pi)^3}\left[u^2_{\mathbf{p}-\mathbf{p}'} + \frac{1}{3}S(S+1)(g^\dagger)^\mu_{\alpha\beta}\sigma^i_{\lambda\alpha}\sigma^i_{\rho\beta}g^\mu_{\lambda\rho}J^2_{\mathbf{p}-\mathbf{p}'} \right]\Gamma_j(\mathbf{p}',\omega)G(\mathbf{p}',\omega)G(-\mathbf{p}',-\omega)$$

Singlet versus Triplet



$$\Gamma_j(\mathbf{p},\omega) = \hat{p}_j + n\int \frac{d^3p'}{(2\pi)^3}\left[u^2_{\mathbf{p}-\mathbf{p}'} + \frac{1}{3}S(S+1)(g^\dagger)^\mu_{\alpha\beta}\sigma^i_{\lambda\alpha}\sigma^i_{\rho\beta}g^\mu_{\lambda\rho}J^2_{\mathbf{p}-\mathbf{p}'}\right]\Gamma_j(\mathbf{p}',\omega)G(\mathbf{p}',\omega)G(-\mathbf{p}',-\omega)$$

singlet

$$\hat{g} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Abrikosov & Gor'kov JETP 1961

$$\tfrac{1}{3}S(S+1)g^t_{\alpha\beta}\sigma^i_{\lambda\alpha}\sigma^i_{\rho\beta}g_{\lambda\rho}J^2_{\mathbf{q}} = -\tfrac{1}{4}S(S+1)J^2_{\mathbf{q}}$$

triplet

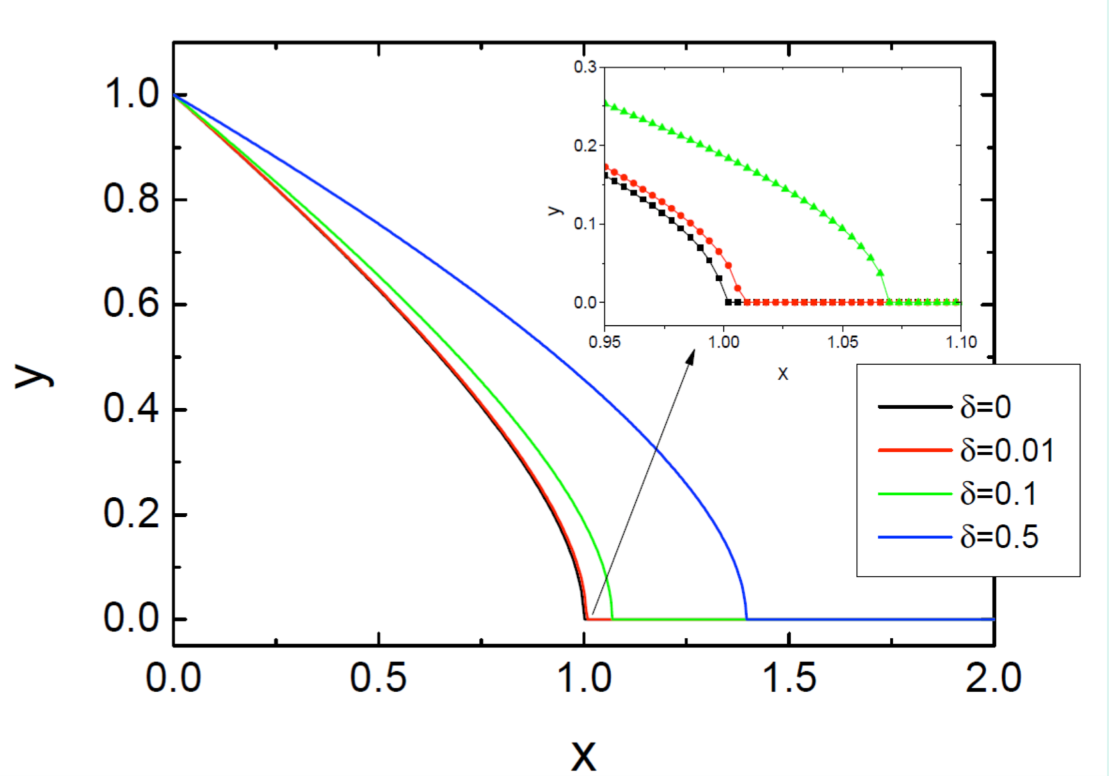
$$g^\nu_{\lambda\rho} = (-\sigma^z_{\lambda\rho}, i\delta_{\lambda\rho}, \sigma^x_{\lambda\rho})$$

$$\tfrac{1}{3}S(S+1)(g^\dagger)^\mu_{\alpha\beta}\sigma^i_{\lambda\alpha}\sigma^i_{\rho\beta}g^\nu_{\lambda\rho}J^2_{\mathbf{q}} = \tfrac{1}{4}S(S+1)J^2_{\mathbf{q}}\delta_{\mu\nu}$$

Critical temperature

$$F_2 = \alpha A^{\star}_{\mu i} A_{\mu i} - 2\eta \left[A^{\star}_{\mu z} A_{\mu z} - \frac{1}{2}(A^{\star}_{\mu x} A_{\mu x} + A^{\star}_{\mu y} A_{\mu y}) \right]$$

$$\alpha = \frac{N_0}{3}\left[\ln\frac{T}{T_{c0}} + \psi\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right) - \psi\left(\frac{1}{2}\right) - \frac{1}{5}\frac{\delta}{4\pi T\tau}\psi^{(1)}\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right)\right]$$



$$\eta = \frac{8N_0}{45}\frac{\delta}{4\pi T\tau}\psi^{(1)}\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right)$$

$$\psi(z) = \frac{d\ln\Gamma(z)}{dz}$$

$$\frac{1}{\tau} = \frac{1}{\tau_p} + \frac{1}{\tau_{ex}}, \qquad \frac{\delta}{\tau} = \frac{\delta_p}{\tau_p} + \frac{\delta_{ex}}{\tau_{ex}}$$

$$y = \frac{T_c}{T_{c0}} = y(x)$$

$$x = \frac{\tau_c}{\tau} = \frac{\gamma}{\pi T_{c0}\tau}$$

$$T_{c1} = T_{co} - \frac{\pi}{8\tau} + \frac{11\pi}{60\tau}\delta$$

Spin current

Residual resistance of a normal metal
Abrikosov&Gor'kov, JETP 1958

$$\mathbf{j}_i = -\frac{\delta H}{\delta \boldsymbol{\omega}_i}$$

$$\boldsymbol{\omega}_i = \nabla_i \boldsymbol{\theta}$$

$$\gamma \mathbf{H} = \frac{\partial \boldsymbol{\theta}}{\partial t} = -i\omega\boldsymbol{\theta}$$

Larmor theorem

$$H = \frac{1}{2m}\int d^3r (D_i^{\alpha\lambda}\psi_\lambda)^\dagger D_i^{\alpha\mu}\psi_\mu + H_{int}$$

$$D_i^{\alpha\beta} = -i\delta_{\alpha\beta}\nabla_i + \frac{1}{2}\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\omega}_i$$

$$\mathbf{j}_i(\mathbf{k},\omega) = \frac{i}{4m} Tr \int_{-\infty}^{+\infty}\frac{d\varepsilon}{2\pi}\int \frac{d^3p}{(2\pi)^3} p_i \boldsymbol{\sigma}_{\alpha\beta}(\boldsymbol{\sigma}_{\beta\alpha}\boldsymbol{\omega}_j)\Pi_j - \frac{1}{4}n\boldsymbol{\omega}_i$$

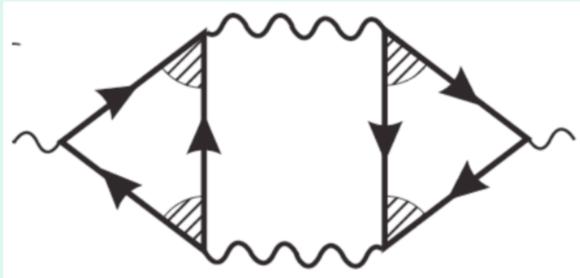
$$\Pi_j(p,p-k) = G(\mathbf{p},\varepsilon)G(\mathbf{p}-\mathbf{k},\varepsilon-\omega)\left[p_j + \int \frac{d^3p'}{(2\pi)^3}U^2(\mathbf{p}-\mathbf{p}')\Pi_j(p',p'-k)\right]$$

$$\mathbf{j}_z^M = -\frac{1}{3}\left\{1 + \frac{16}{15}\delta\right\}\tau v_F^2 \nabla_z \mathbf{M}$$

$$\mathbf{j}_x^M = -\frac{1}{3}\left\{1 - \frac{8}{15}\delta\right\}\tau v_F^2 \nabla_x \mathbf{M}$$

$$\frac{1}{\tau} = \frac{1}{\tau_p} + \frac{1}{\tau_{ex}}, \qquad \frac{\delta}{\tau} = \frac{\delta_p}{\tau_p} + \frac{\delta_{ex}}{\tau_{ex}}$$

Paradiffusion



Aslamazov & Larkin, 1968

$$\mathbf{j}_i(\omega_\nu) = \int \frac{d^3q}{(2\pi)^3} T \sum_k \mathbf{B}^{lr}_{i,\alpha\beta}(\mathbf{q},\Omega_k,\omega_\nu) L^{rs}(\mathbf{q},\Omega_k)(\mathbf{B}^{st}_{j,\beta\alpha}(\mathbf{q},\Omega_k,\omega_\nu)\cdot\boldsymbol{\omega}_j) L^{tl}(\mathbf{q},\Omega_k+\omega_\nu)$$

$$\mathbf{B}^{lr}_{i,\alpha\beta}(\mathbf{q},\Omega_k,\omega_\nu) = T\sum_n \int \frac{d^3p}{(2\pi)^3} v_i \boldsymbol{\sigma}_{\alpha\beta} \Lambda^l_{\mathbf{q},\varepsilon_n+\omega_\nu,\Omega_k-\varepsilon_n} \Lambda^{r\star}_{\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n} G_{\mathbf{p},\varepsilon_n+\omega_\nu} G_{\mathbf{p},\varepsilon_n} G_{\mathbf{q}-\mathbf{p},\Omega_k-\varepsilon_n}$$

$$\Lambda^l_{\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n} = \hat{p}_l + \frac{1}{2\pi N_o\tau}\int \frac{d^3k}{(2\pi)^3} G_{\mathbf{k},\varepsilon_n} G_{-\mathbf{k}+\mathbf{q},\Omega_k-\varepsilon_n} \Lambda^l_{\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n}$$

$$L^{rs}(\mathbf{q},\Omega_k) = \left(\frac{1}{3g}\delta_{rs} - T\sum_n \int \frac{d^3k}{(2\pi)^3} G_{\mathbf{k},\varepsilon_n} G_{-\mathbf{k}+\mathbf{q},\Omega_k-\varepsilon_n} \hat{k}_r \Lambda^s_{\mathbf{q},\varepsilon_n}\right)^{-1}$$

$$\mathbf{j}^{fl}_i = -\frac{45}{8\xi\sqrt{\epsilon}}\mu\nabla_i\mathbf{H}$$

$$\mathbf{j}^{dif}_i = -\hbar N_0 D\mu\nabla_i\mathbf{H}$$

$$D = \tfrac{1}{3}\tau v_F^2$$

$$\frac{j^{fl}}{j^{dif}} = \frac{45}{8\hbar\xi N_0 D}\frac{1}{\sqrt{\epsilon}} \approx 5\times10^{-2}\frac{1}{\sqrt{\epsilon}}$$

$$\epsilon = \ln\frac{T}{T_c}$$

Conclusion

The anisotropy suppression in anisotropic aerogel filled by pure He-3 is determined by the sum of two independent terms originating from the potential and the exchange scattering.

The change of anisotropy of scattering at different types of covering must also reveal itself in the changes of spin diffusion anisotropy.

$$\frac{\delta}{\tau}_{pure\ \ He-3} = \frac{\delta_p}{\tau_p}_{pure\ \ He-3} + \frac{\delta_{ex}}{\tau_{ex}}_{pure\ \ He-3} << \frac{\delta_p}{\tau_p}_{preplated}$$

$$\frac{D_z}{D_x} = \frac{1 + \frac{16}{15}\delta}{1 - \frac{8}{15}\delta}$$

VM, PRB 98, 014501 (2018)
arXiv:1904.03899