Some Many-body Phenomena due to Quantum and Quasi-classical Effects of Electron Motion along Open Orbits in a Magnetic Field

Modern Trends in Condensed Matter Physics
(Lev P. Gor’kov’s Memorial Conference, September 25, 2019; Chernogolovka, Russian Federation)

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The Gor'kov's equations

$$\left\{i\frac{\partial}{\partial t}+\frac{1}{2m}\left(\frac{\partial}{\partial \mathbf{r}}-ie\mathbf{A}(\mathbf{r})\right)^2+\mu\right\}G(x,x')$$

$$+i\Delta(\mathbf{r})F^{+}(x,x')=\delta(x-x'),$$

$$\left\{i\frac{\partial}{\partial t}-\frac{1}{2m}\left(\frac{\partial}{\partial \mathbf{r}}+ie\mathbf{A}(\mathbf{r})\right)^2-\mu\right\}F^{+}(x,x')$$

$$-i\Delta^*(\mathbf{r})G(x,x')=0,$$

$$\Delta^*(\mathbf{r})=|g|F^{+}(x,x),$$

Normal and anomalous Green's functions

Gor'kov's gap equation in a magnetic field

$$\Delta^*(\mathbf{r}) = -i|g|\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega \int \widetilde{G}_\omega(\mathbf{s},\mathbf{r})\ \widetilde{G}'_{-\omega}(\mathbf{s},\mathbf{r})\,\Delta^*(\mathbf{s})\,d^3\mathbf{s}. \tag{6}$$

$$\widetilde{G}_\omega(\mathbf{r},\mathbf{r}') = \exp\left\{-\frac{ieH}{2c}(y+y')(x-x')\right\}\widetilde{G}'_\omega(\mathbf{r}-\mathbf{r}').$$

$$\left\{\omega + \frac{1}{2m}\left(\frac{\partial}{\partial\mathbf{r}} - \frac{ie}{2c}\mathbf{H}\times[\mathbf{r}-\mathbf{r}']\right)^2 + \mu\right\}\widetilde{G}'_\omega(\mathbf{r}-\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'). \tag{8}$$

$$G_{0\omega}(\mathbf{R}) = \begin{cases} -(m/2\pi R)\exp[ip_0R + i\omega R/v], & \omega > 0 \\ -(m/2\pi R)\exp[-ip_0R + i\omega R/v], & \omega < 0 \end{cases} \quad (|\omega| \ll \mu). \tag{9}$$

Upper critical magnetic field at T=0

$$\Delta^*(y)\ln\left(\frac{e\gamma\Delta_0}{v}\varepsilon\right) = -\frac{1}{2}\int_{-\infty}^{\infty}\frac{\exp\{-|(eH/c)(y^2-y'^2)|\}}{|y-y'|}\Delta^*(y')\,d_\varepsilon y'.$$

$$\Delta^*(\xi)\ln\left(\frac{\Delta_0 e}{v}\sqrt{\frac{\gamma c}{eH}}\right) = \Phi(\xi)\Delta^*(\xi) - \frac{1}{2}\int_{-\infty}^{\infty}\frac{\exp\{-|\xi^2-\xi'^2|\}}{|\xi-\xi'|}[\Delta^*(\xi')-\Delta^*(\xi)]\,d\xi'.$$

$$\Phi(\xi) = e^{-\xi^2}\int_0^{|\xi|} e^{\xi'^2}\xi'\ln\frac{|\xi|+\xi'}{|\xi|-\xi'}\,d\xi'.$$

$$H_{c1} \approx (e^2\gamma/2)(c\Delta_0^2/ev).$$

Variation method with the correct trial function

Layered anisotropic superconductor

$$\epsilon(\mathbf{p}) = \frac{1}{2m}(p_x^2 + p_y^2) - 2t_\perp \cos(p_z c^*) \ ,$$

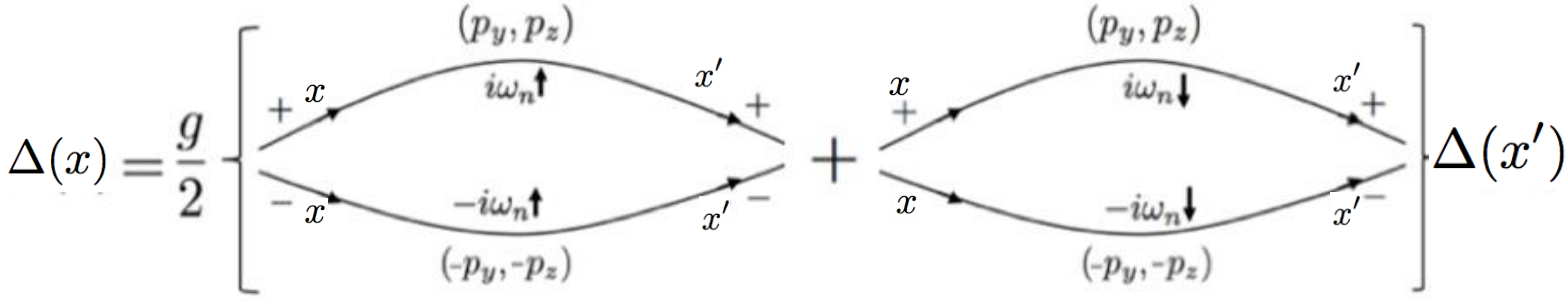
$$t_\perp \ll \epsilon_F \ , \qquad \epsilon_F = \frac{p_F^2}{2m} = \frac{m v_F^2}{2} \ ,$$

$$g^{\pm}_{i\omega_n}(x, x_1; p_y, p_z; \sigma) = -i \frac{sgn(\omega_n)}{v_x(p_y)} \exp\left[\mp \frac{\omega_n (x - x_1)}{v_x(p_y)}\right]$$
$$\times \exp\left[\pm i \frac{2t_\perp \cos(p_z c^*)(x - x_1)}{v_x^0}\right] \exp\left[\frac{\mp i \mu_B \sigma H (x - x_1)}{v_x(p_y)}\right]$$
$$\times \exp\left[\pm i \frac{t_\perp \omega_c (x^2 - x_1^2)}{v_x^0 v_F} \sin(p_z c^*)\right]. \qquad (15)$$

- A.G. Lebed, PRB (2019)

tc >> Tc : MgB2

Gap Equation in a Magnetic Field



- Integral equation for superconducting nucleus:

$$\Delta(x) = g \int K(x, x')\Delta(x')$$

- Kernel:

$$K(x, x') = T \sum_{\omega_n} \left\langle G^{+}_{i\omega_n}(x, x'; p_y, p_z) G^{-}_{-i\omega_n}(x, x'; -p_y, -p_z) \right\rangle$$

Ginzburg-Landau equation

$$-\xi_{\parallel}^2 \frac{d^2\Delta(x)}{dx^2} + \left(\frac{2\pi H}{\phi_0}\right)^2 \xi_{\perp}^2 x^2 \Delta(x) - \tau\Delta(x) = 0,$$

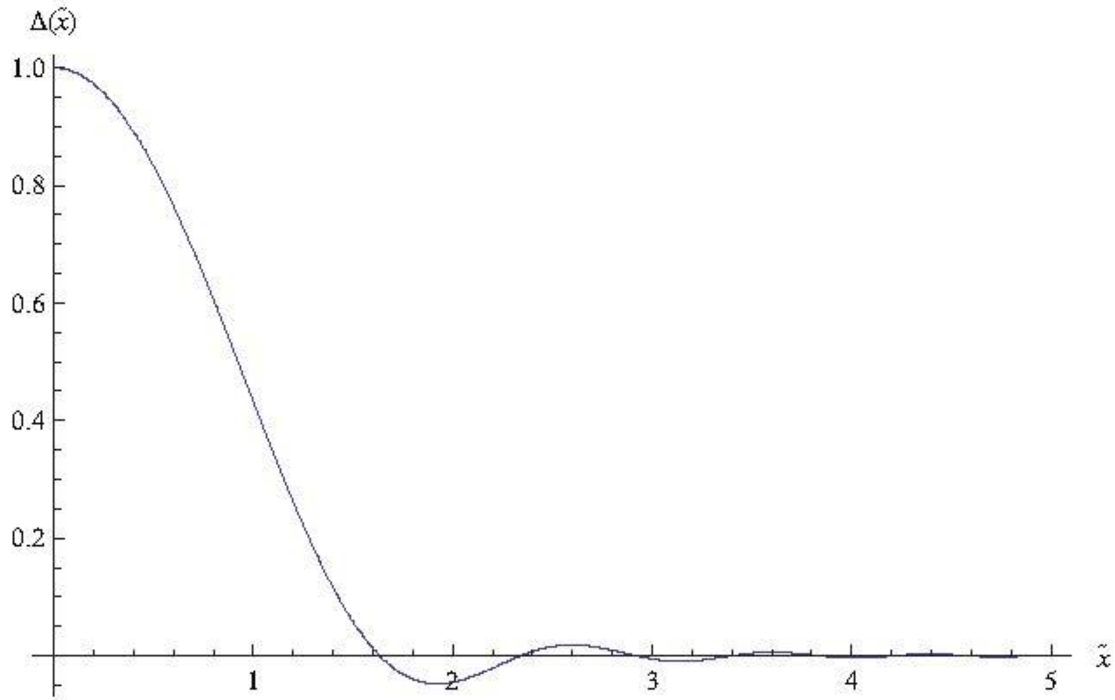
$$H_{c2}^{\parallel} = \tau\left(\frac{\phi_0}{2\pi\xi_{\parallel}\xi_{\perp}}\right) = \tau\left[\frac{8\pi^2 c T_c^2}{7\zeta(3) e v_F t_{\perp} c^*}\right].$$

$$H_{c2}^{\perp} = \tau\left(\frac{\phi_0}{2\pi\xi_{\parallel}^2}\right) = \tau\left[\frac{16\pi^2 c T_c^2}{7\zeta(3) e v_F^2}\right].$$

Takafumi Kita, PRB (2003)

Gor’kov’s integral equation at T=0

$$\Delta(x) = g\left\langle \int_d^\infty \frac{dz}{z}\; J_0\left\{\frac{2t_\perp \omega_c}{v_F^2}[z(2x + z\sin\alpha)]\right\} \times \cos\left[\frac{2\mu_B H z}{v_F}\right]\; \Delta(x + z\sin\alpha)\right\rangle_\alpha . \quad (25)$$



Anisotropy changes

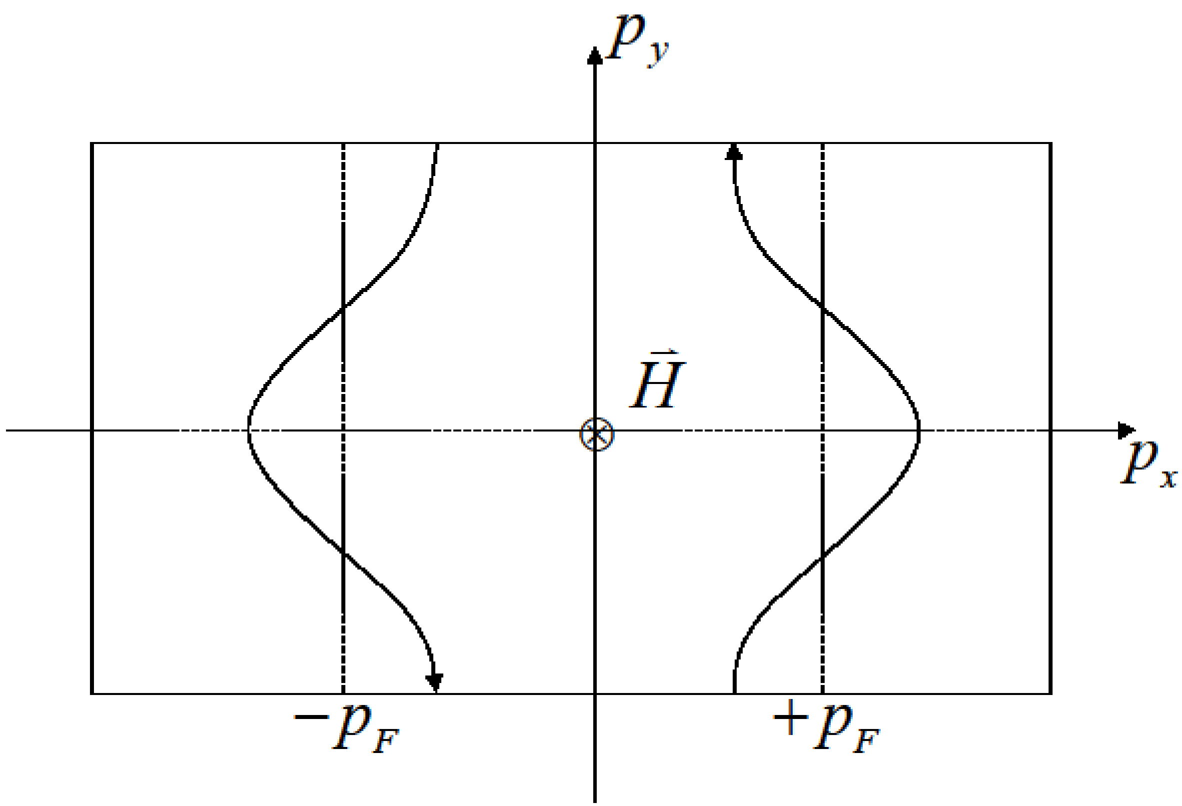
$$H_{c2}^{\parallel}(0) \approx 0.75 \left[\frac{8\pi^2 c T_c^2}{7\zeta(3) e v_F t_\perp c^*} \right] = 0.75 \ |dH_{c2}^{\parallel}/dT|_{T_c} T_c$$

$$H_{c2}^{\perp}(0) \approx 0.59 \left[\frac{16\pi^2 c T_c^2}{7\zeta(3) e v_F^2} \right] = 0.59 \ |dH_{c2}^{\perp}/dT|_{T_c} T_c$$

$$\lim_{T\to 0} \gamma(T) = \lim_{T\to 0} \left[\frac{H_{c2}^{\parallel}(T)}{H_{c2}^{\perp}(T)} \right] = 1.27 \lim_{T\to T_c} \gamma(T).$$

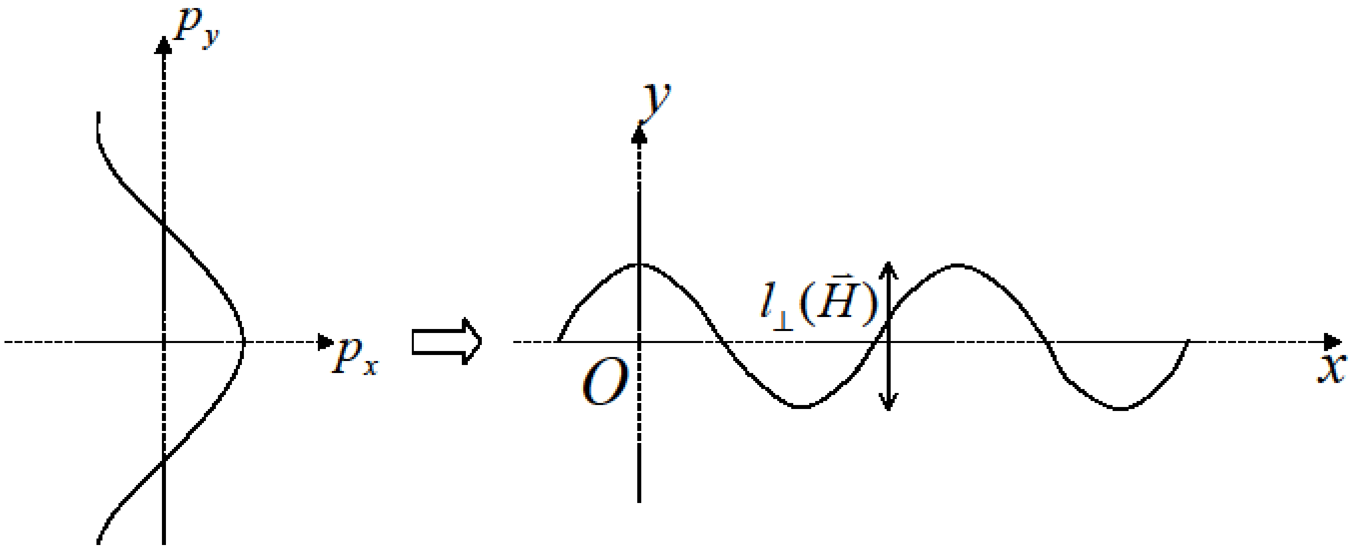
- Effective mass model is broken

Quantum effects of electron motion along open Q1D Fermi surfaces

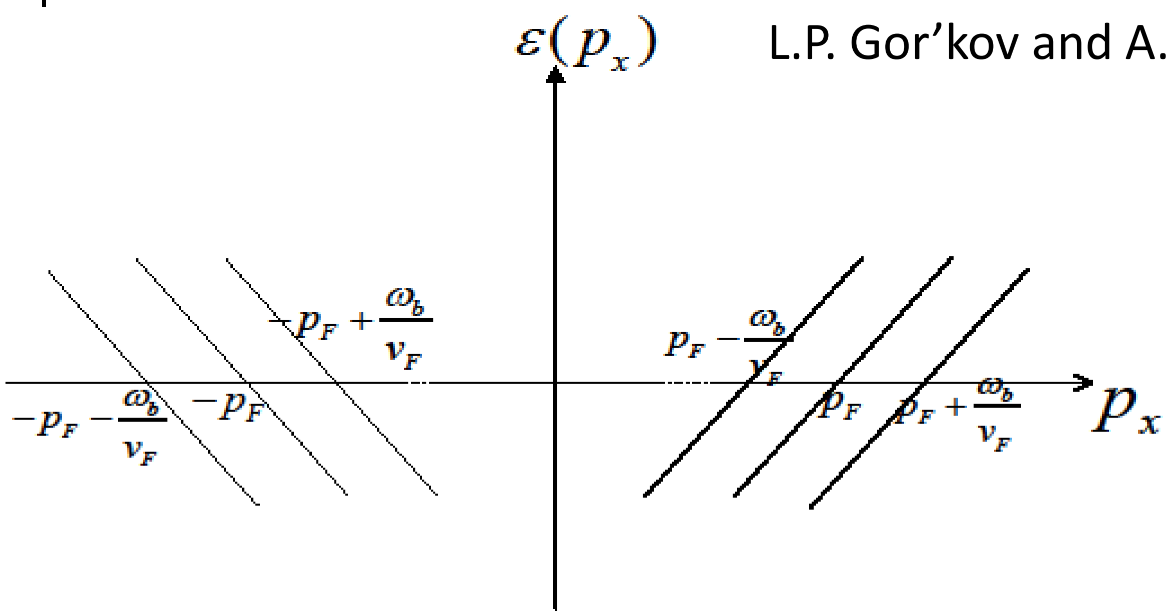


$$\frac{d\boldsymbol{p}}{dt} = \frac{e}{c}[\mathbf{v} \times \boldsymbol{H}]$$

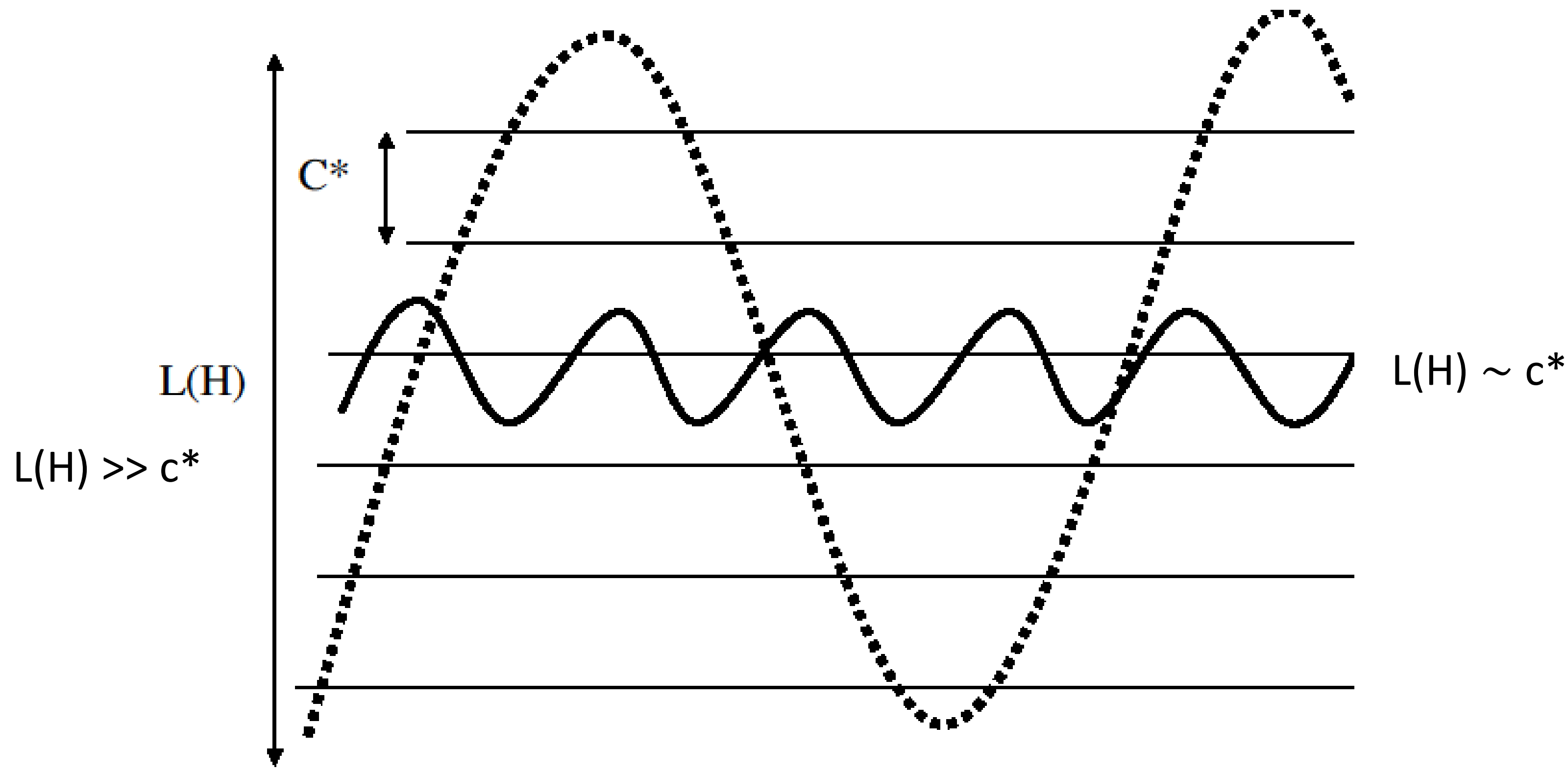
Motion in a real space; one-dimensionalization of Q1D electron spectrum



L.P. Gor’kov and A.G. Lebed (1984)

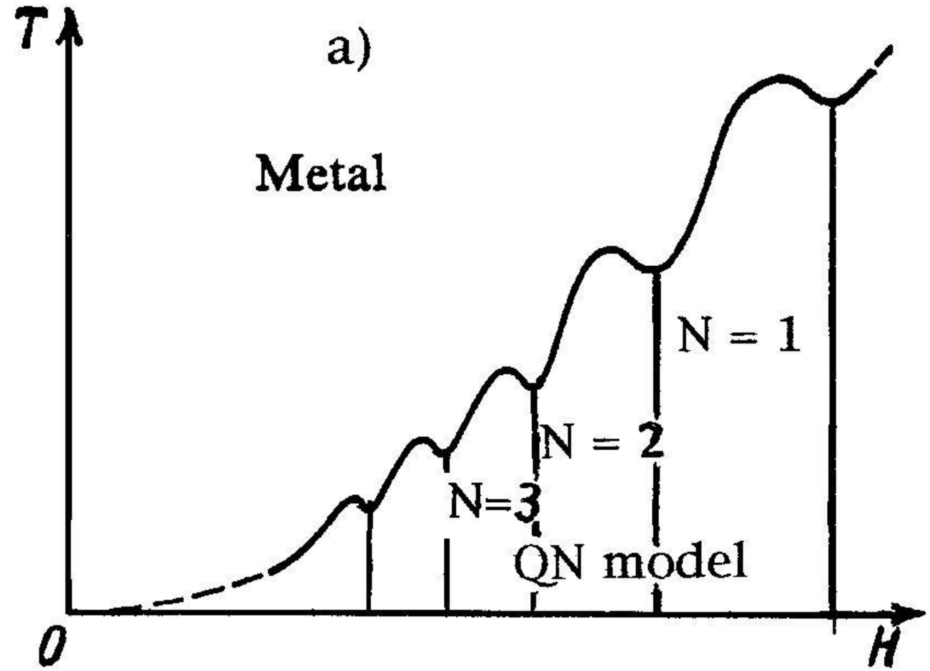


Quantum limit in a magnetic field



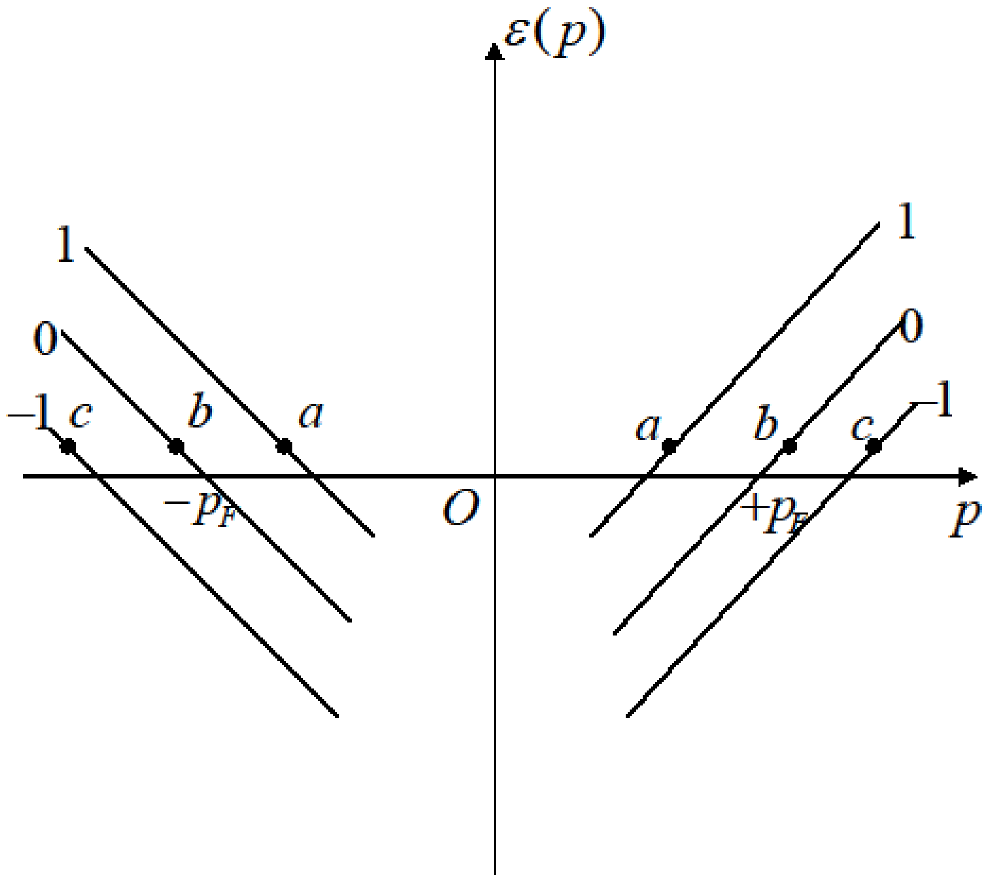
A.G. Lebed (1986)

Quantized Nesting Model

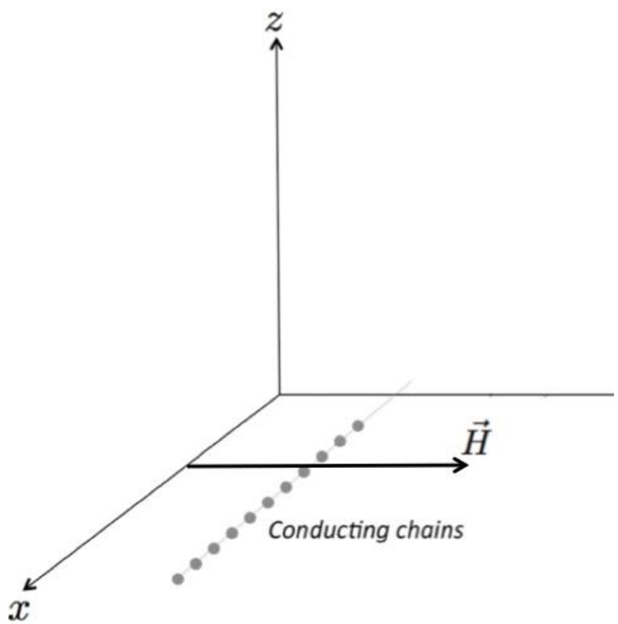


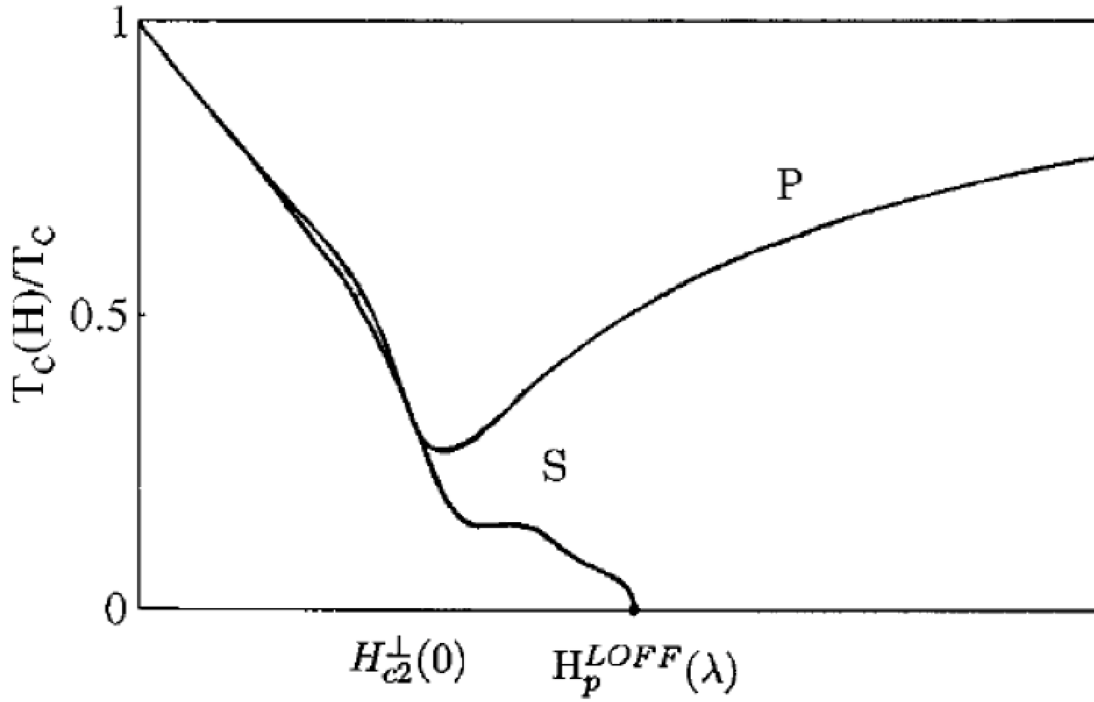
L.P. Gor’kov and A.G. Lebed (1984)
M. Heritier, G. Montambaux, and P. Lederer (1984)
A.G. Lebed (1985, 2002)
K. Maki (1986)
V.M. Yakovenko (1991)

Cooper instability



Reentrant Superconductivity!



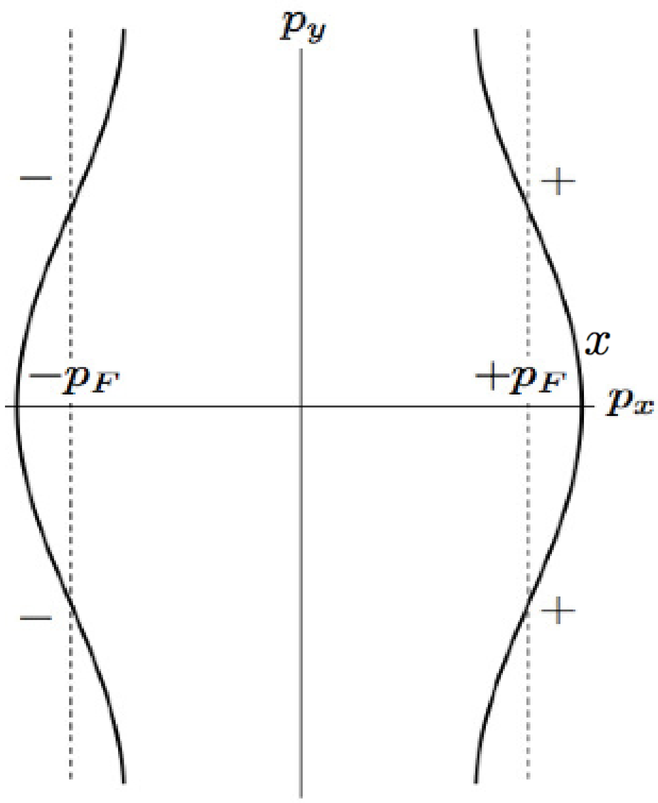


Q1D case: A.G. Lebed, JETP Letters (1986)

Q2D case: A.G. Lebed and K. Yamaji, PRL (1998)

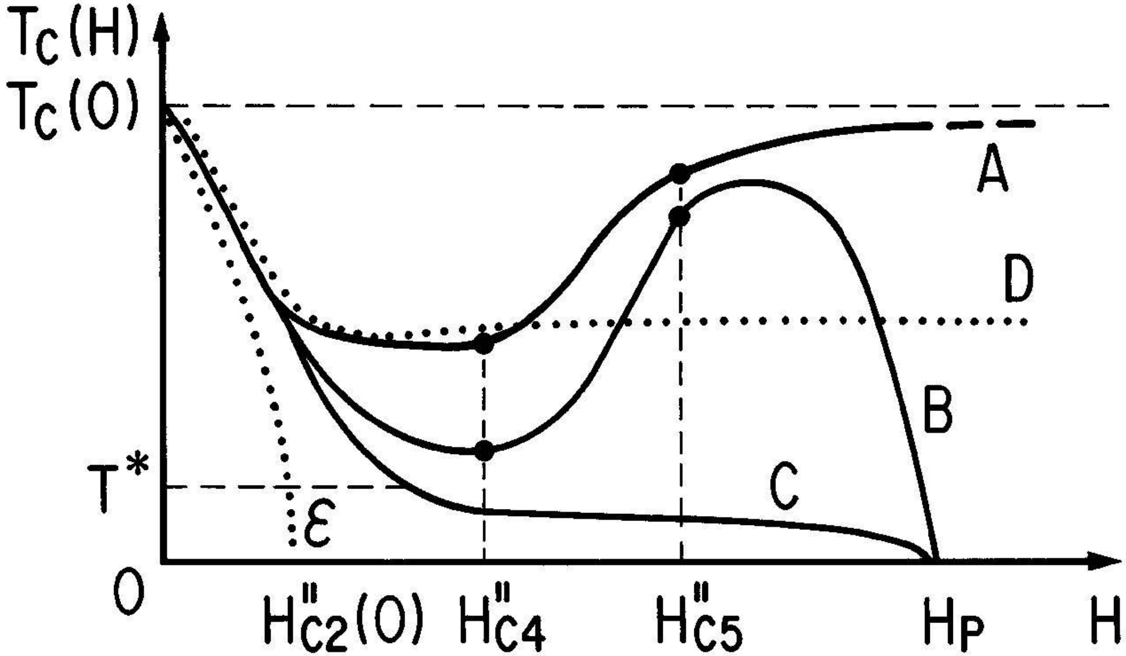
Triplet superconducting order parameter:

Q1D Fermi
Surface



$$\hat{\Delta}(p_x, x) = \hat{I}\ sgn(p_x)\ \Delta(x)$$

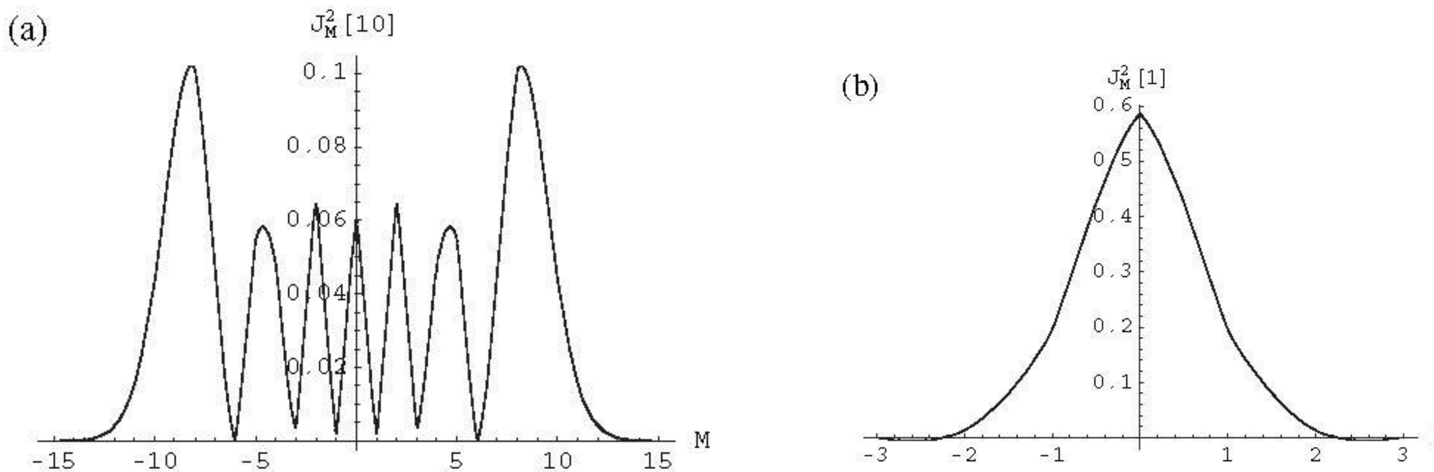
Cooper instability: Q2D case



$$\Delta(\phi, x) = \int_0^{2\pi} \frac{d\phi_1}{2\pi} U(\phi, \phi_1) \int_{|x-x_1|>a^*|\sin\phi_1|}^{\infty} dx_1 \frac{2\pi T}{v_F \sin\phi_1 \sinh[(2\pi T|x - x_1|/v_F \sin\phi_1)]}$$

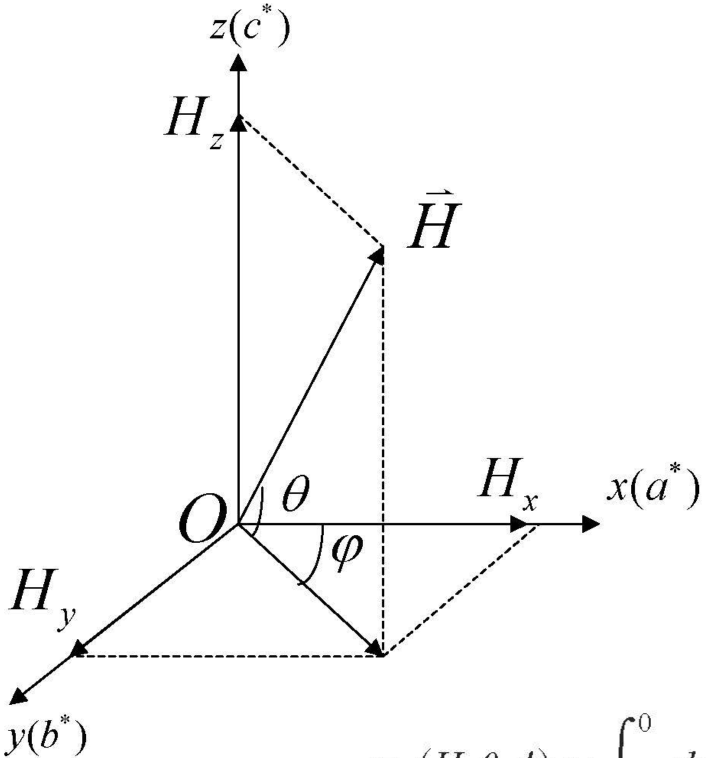
$$\times J_0\left\{\frac{2\lambda}{\sin\phi_1} \sin\left[\frac{\omega_c(x - x_1)}{2v_F}\right] \sin\left[\frac{\omega_c(x + x_1)}{2v_F}\right]\right\} \cos\left[\frac{2k\mu_B H(x - x_1)}{v_F \sin\phi_1}\right] \Delta(\phi_1, x_1)\,.$$

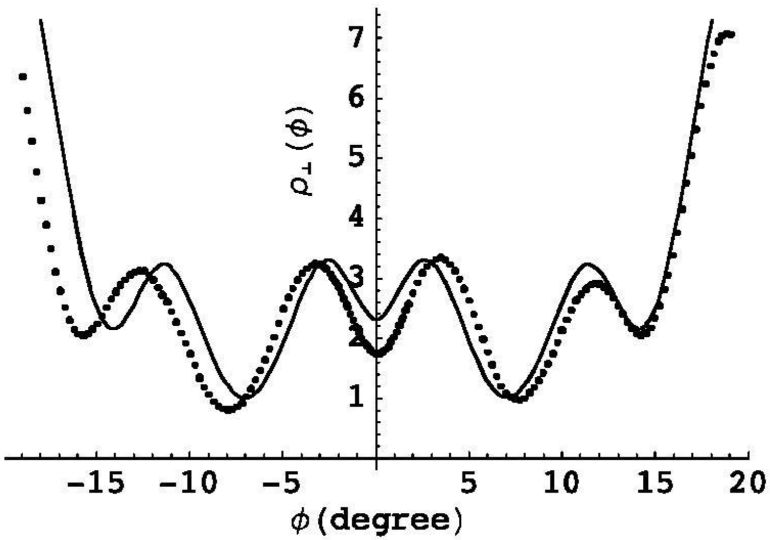
Exact solution vs. Peierls substitution



- A.G. Lebed, PRL (2005)

Lebed and Lee-Naughton-Lebed Magic Angles

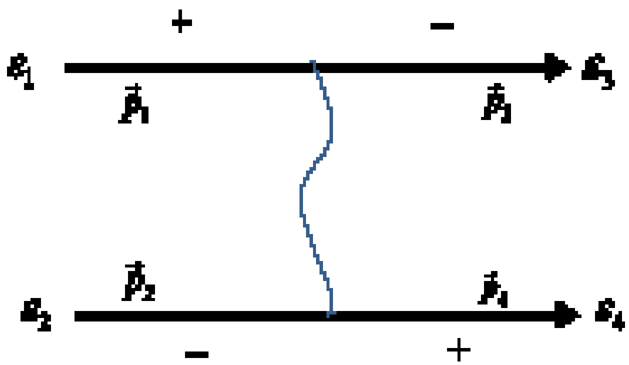




$$\sigma_\perp(H,\theta,\phi) \sim \int_{-\infty}^{0} dz \exp(z) \int_0^{2\pi} \frac{dy}{2\pi} \cos\left[\omega_c(\theta,\phi)\tau z + a\left(\frac{\cos\phi}{\tan\theta}\right)\{f[\omega_b(\theta)\tau z + y] - f[y]\} \right].$$

J.I. Lee, M.J. Naughton, PRB (1998); A.G. Lebed, M.J. Naughton, PRL (2002)

Non-Fermi-liquid scattering



$$\frac{1}{\tau} \sim g^2 T \sim T\,.$$

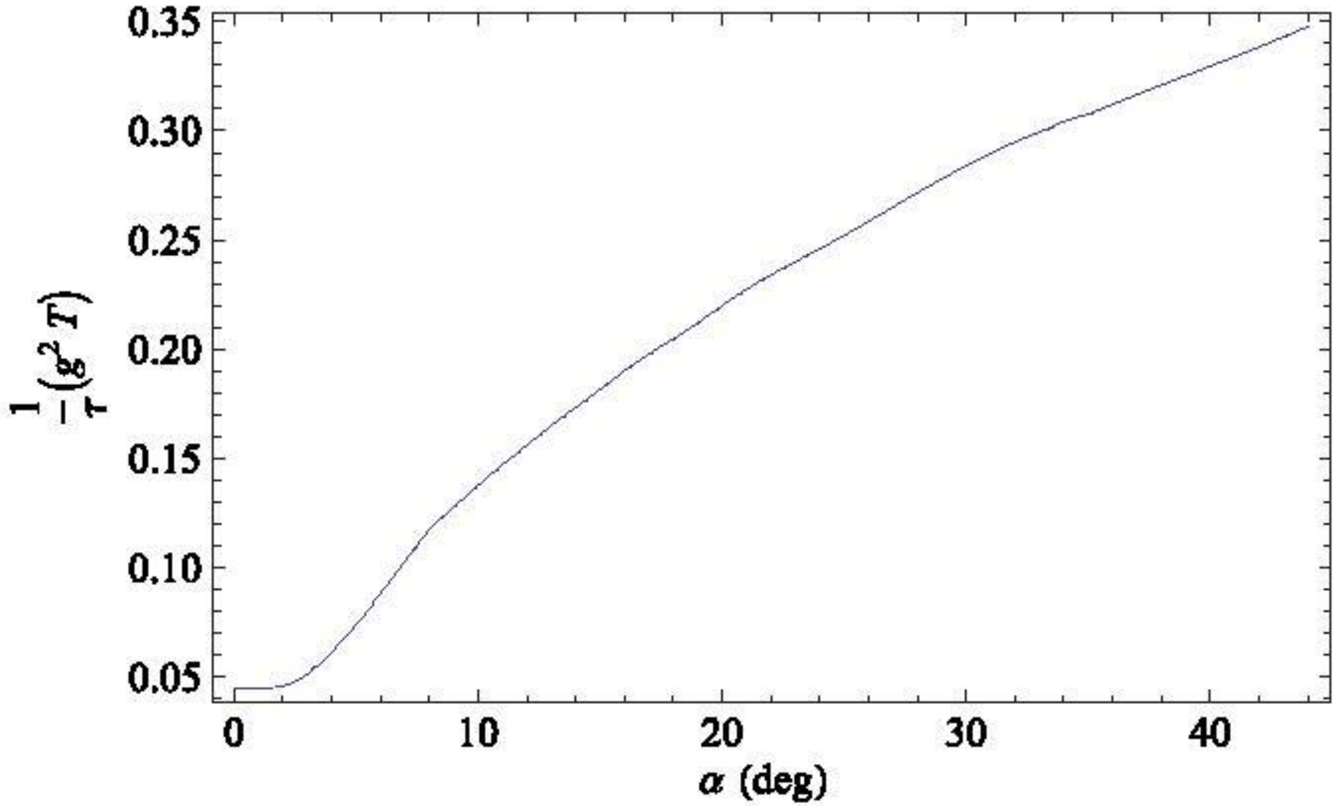
$$\frac{1}{\tau}(0^0) \simeq \frac{g^2T^2}{2\pi t_y}\ln^2\left(\frac{t_y}{T}\right) \ll T\,.$$

$$\frac{1}{\tau}(90^0) \simeq \frac{g^2T^2}{2\pi t_z}\ln^2\left(\frac{t_z}{T}\right) \ll T$$

$$\frac{1}{\tau} = 2g^2T\int_0^\infty \left(\frac{2\pi T\,dx}{v_F}\right)\left[\frac{\left(\frac{2\pi T x}{v_F}\right)\cosh\left(\frac{2\pi T x}{v_F}\right) - \sinh\left(\frac{2\pi T x}{v_F}\right)}{\sinh^3\left(\frac{2\pi T x}{v_F}\right)}\right]$$
$$\times \left\langle J_0^2\left\{4l_y(\alpha)\sin\left[\frac{\omega_y(\alpha)x}{2v_F}\right]\cos(\phi_1)\right\}\right\rangle_{\phi_1}$$
$$\times \left\langle J_0^2\left\{4l_z(\alpha)\sin\left[\frac{\omega_z(\alpha)x}{2v_F}\right]\cos(\phi_2)\right\}\right\rangle_{\phi_2}, \qquad (16)$$

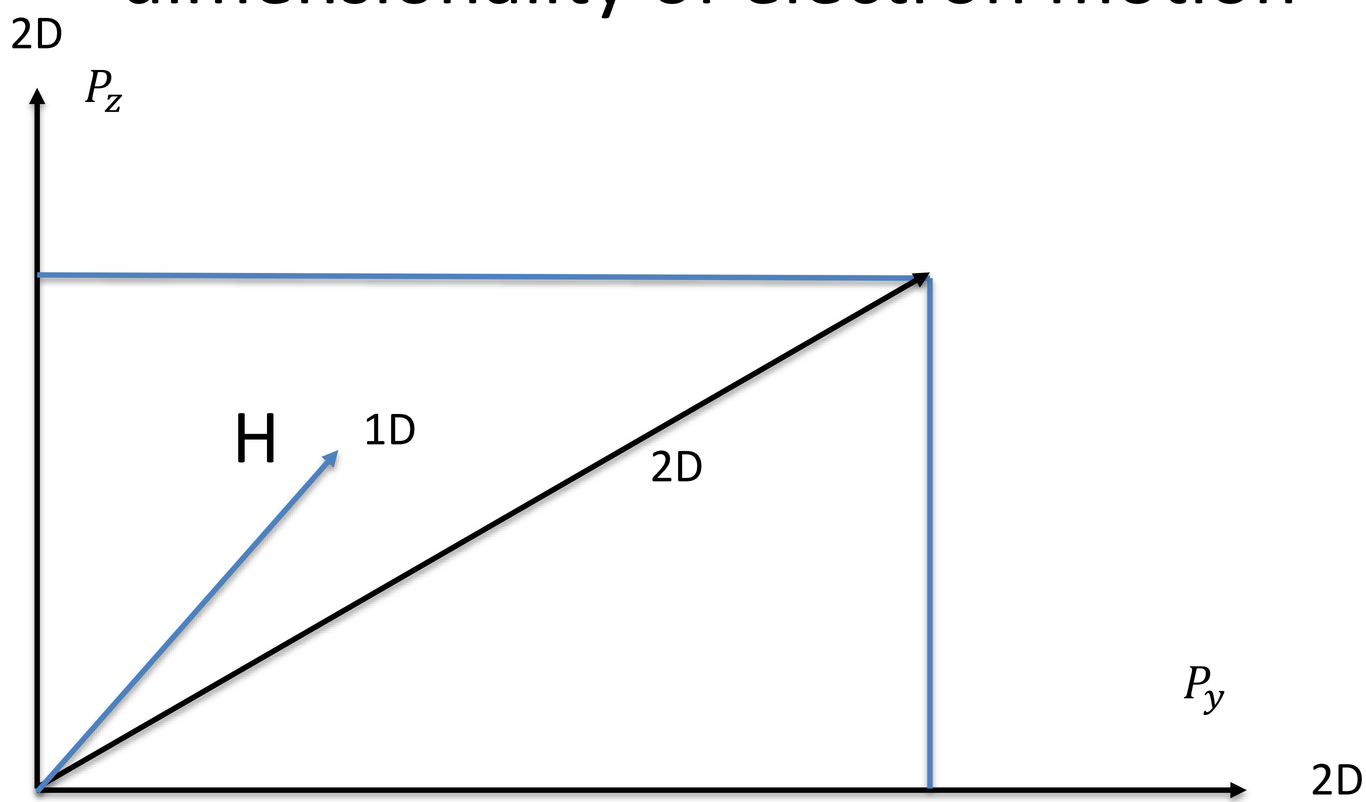
Real calculations for (Per)2Au(mnt)2

$t_y \sim 20K \qquad t_z \sim 2K$



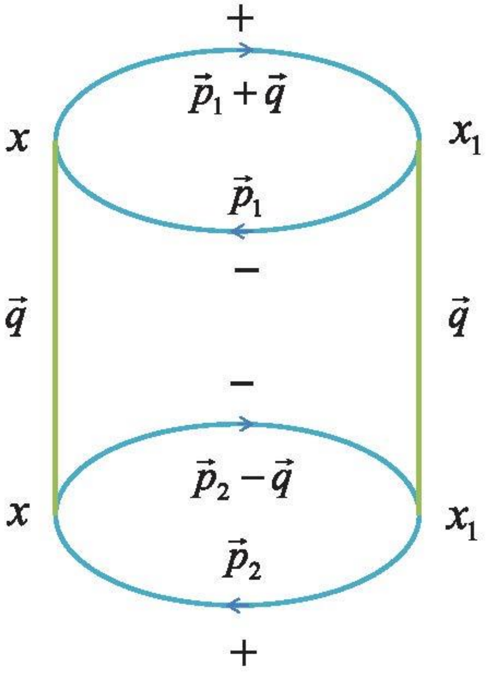
A.G. Lebed, PRL (2015)

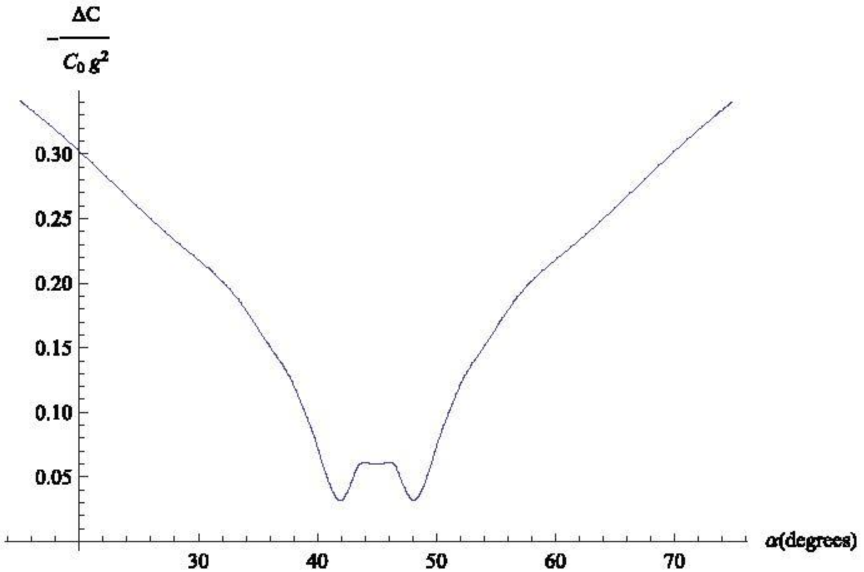
Lebed's Magic Angles: a change of dimensionality of electron motion



G.S. Boebinger (1990); M.J. Naughton (1991); T. Osada (1991)

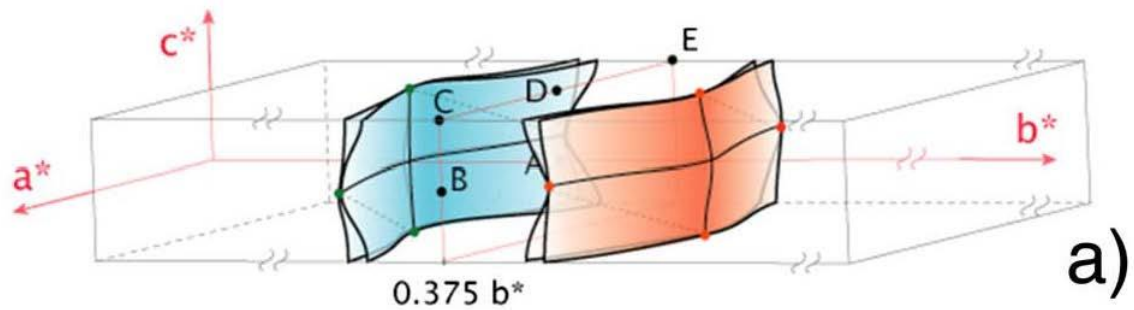
Dimensional Crossovers in Thermodynamic properties

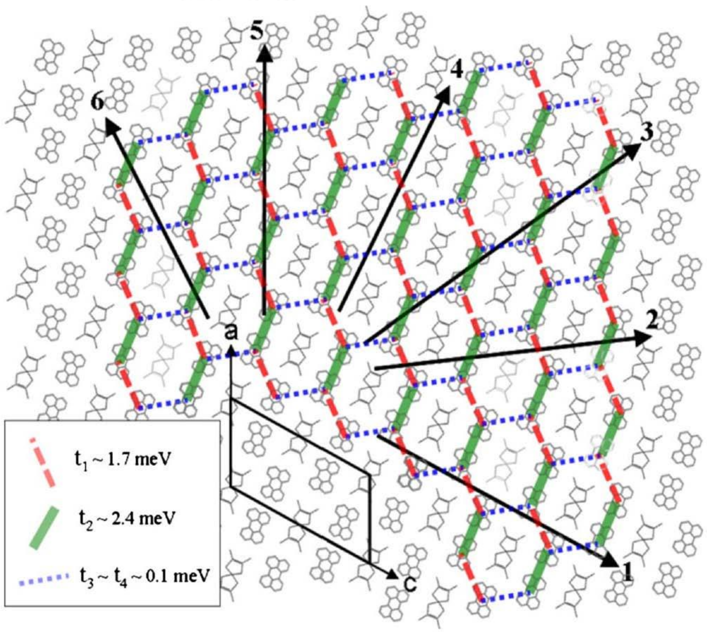




- A.G. Lebed, PRB (2016)

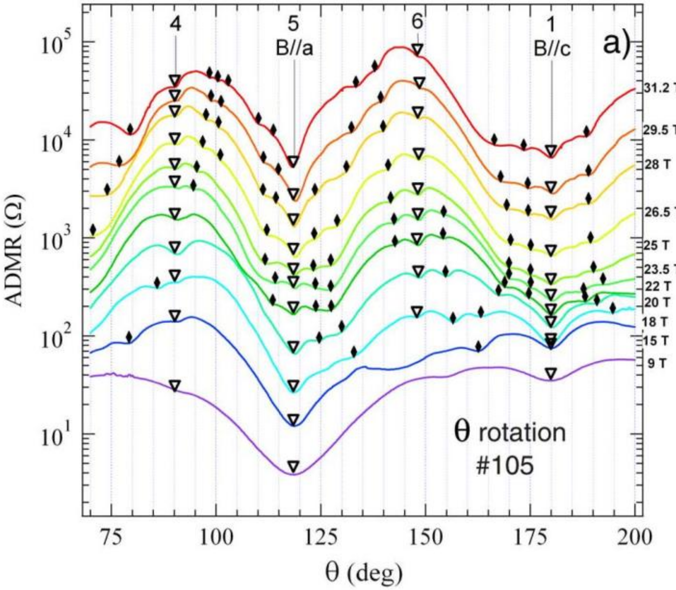
Real experiments on (Per)2Au(mnt)2

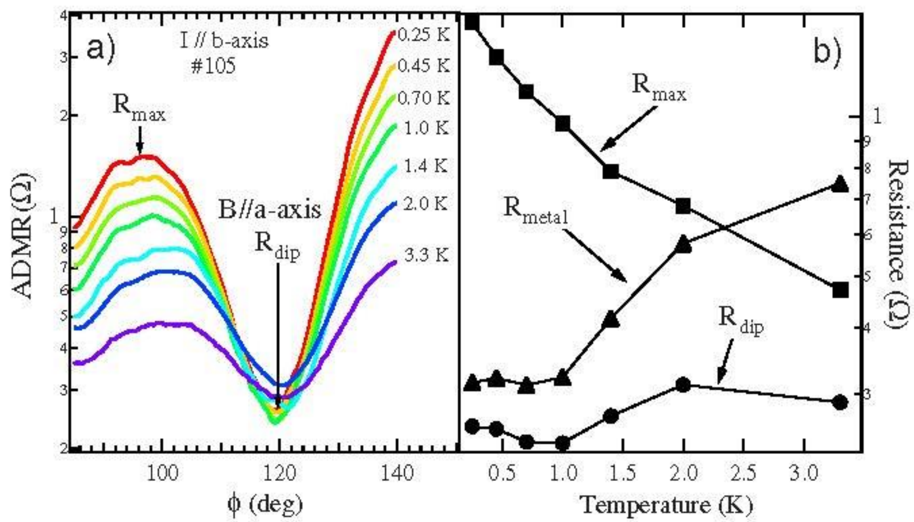




- D. Graf, J.S. Brooks, et al., PRL (2009) at the NHMFL

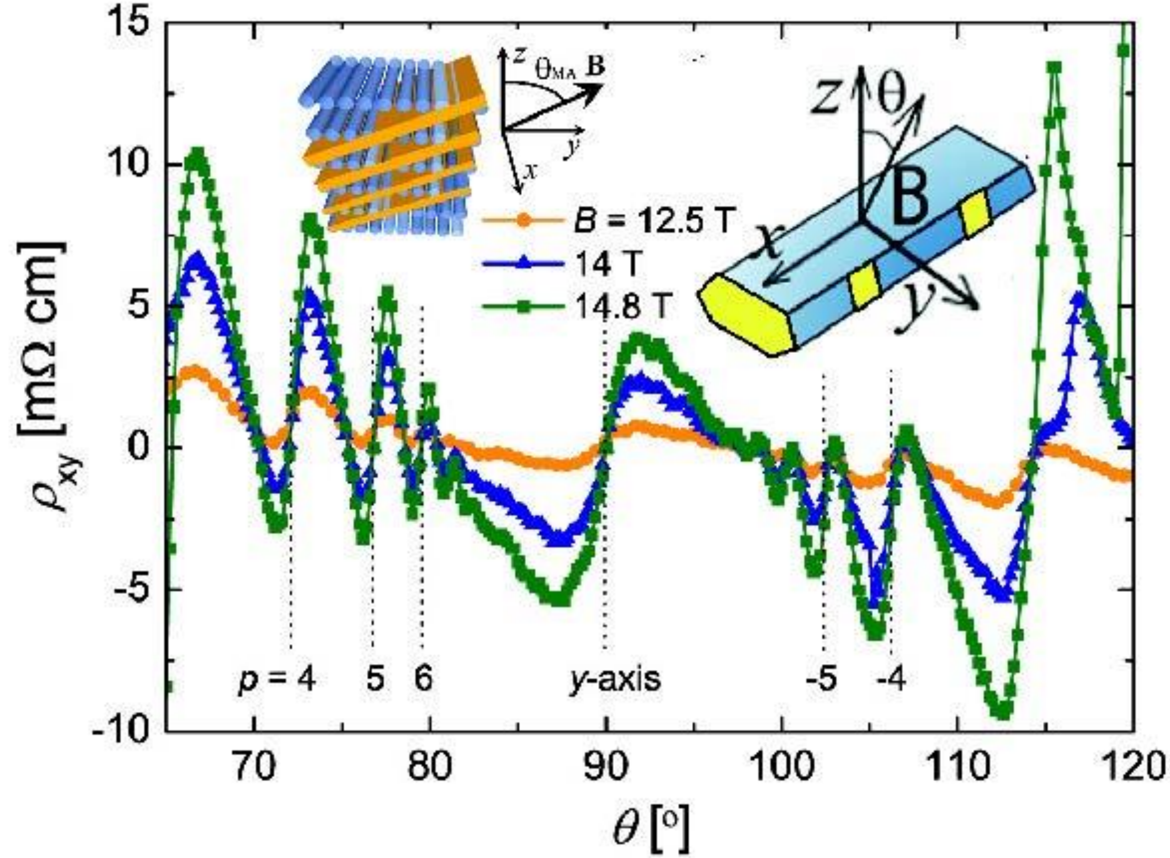
Results of Graf-Brooks (NHMFL) experiments





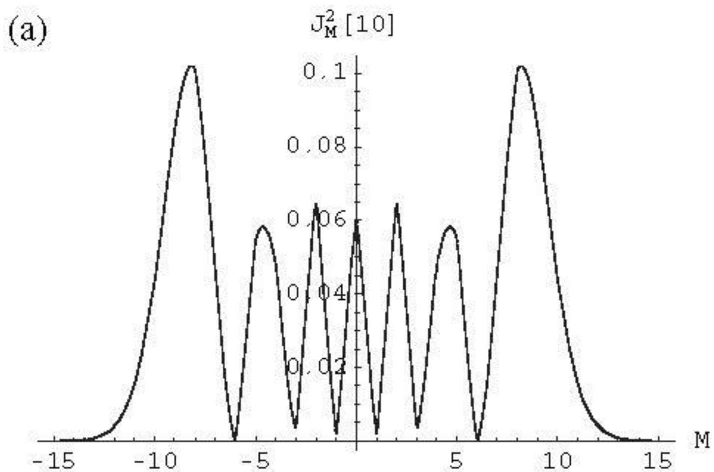
Is it real FL-n-FL angle transitions?

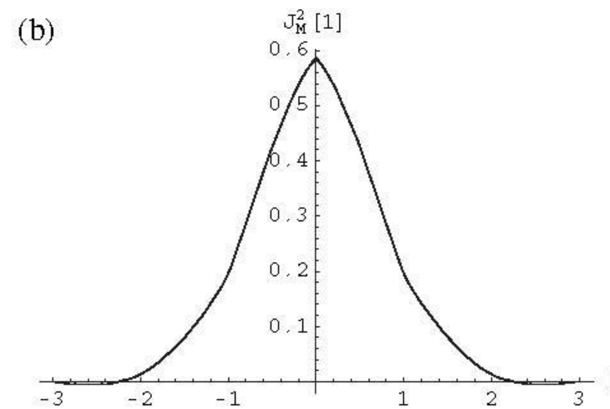
Completely unexplained results



- Kaya Kobayashi, et al., PRL (2014): $(TMTSF)_2ClO_4$

Exact solution vs. Peierls substitution





- A.G. Lebed, PRL (2005, 2017)