Международная конференция
посвящённая 90-летию Исаака Марковича Халатникова

Черноголовка, Московская обл., Россия, 22-23, Октябрь 2009

On the Stochasticity in Relativistic Cosmology

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Phase diagram for diffusion limited aggregation growth in two dimensions

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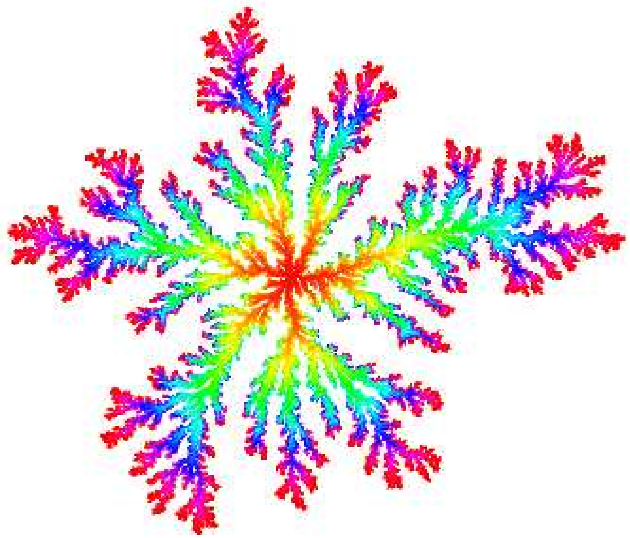
Landau Institute for Theoretical Physics
Chernogolovka, Russia

Problem

- Understanding critical properties of the dynamical processes
- Of particular interest - the critical properties of the dynamical growth
- Present research - the random fractal grows in 2D, dominated by diffusion processes



2D aggregate growth (geometrical critical phenomena)

Ice crystals

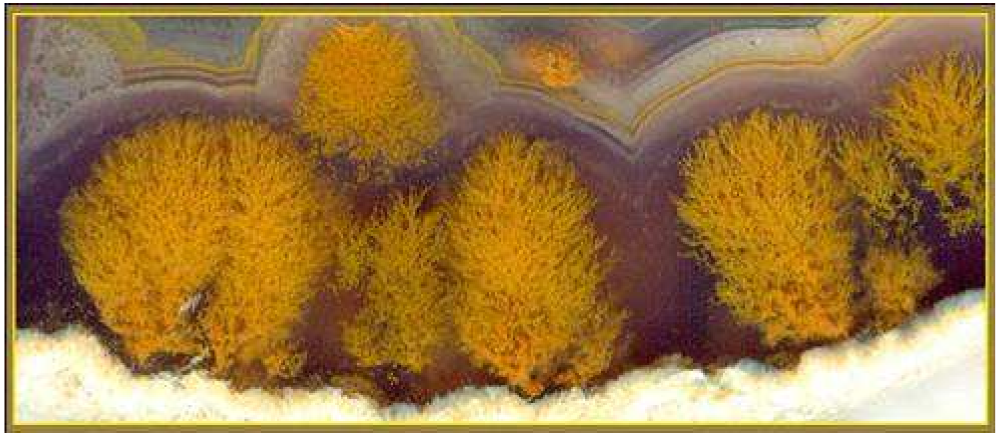
D_3=2.2-2.6

D_2=1.4-1.8

2D aggregate growth (geometrical critical phenomena)

Dendrits

D=1.5-1.8

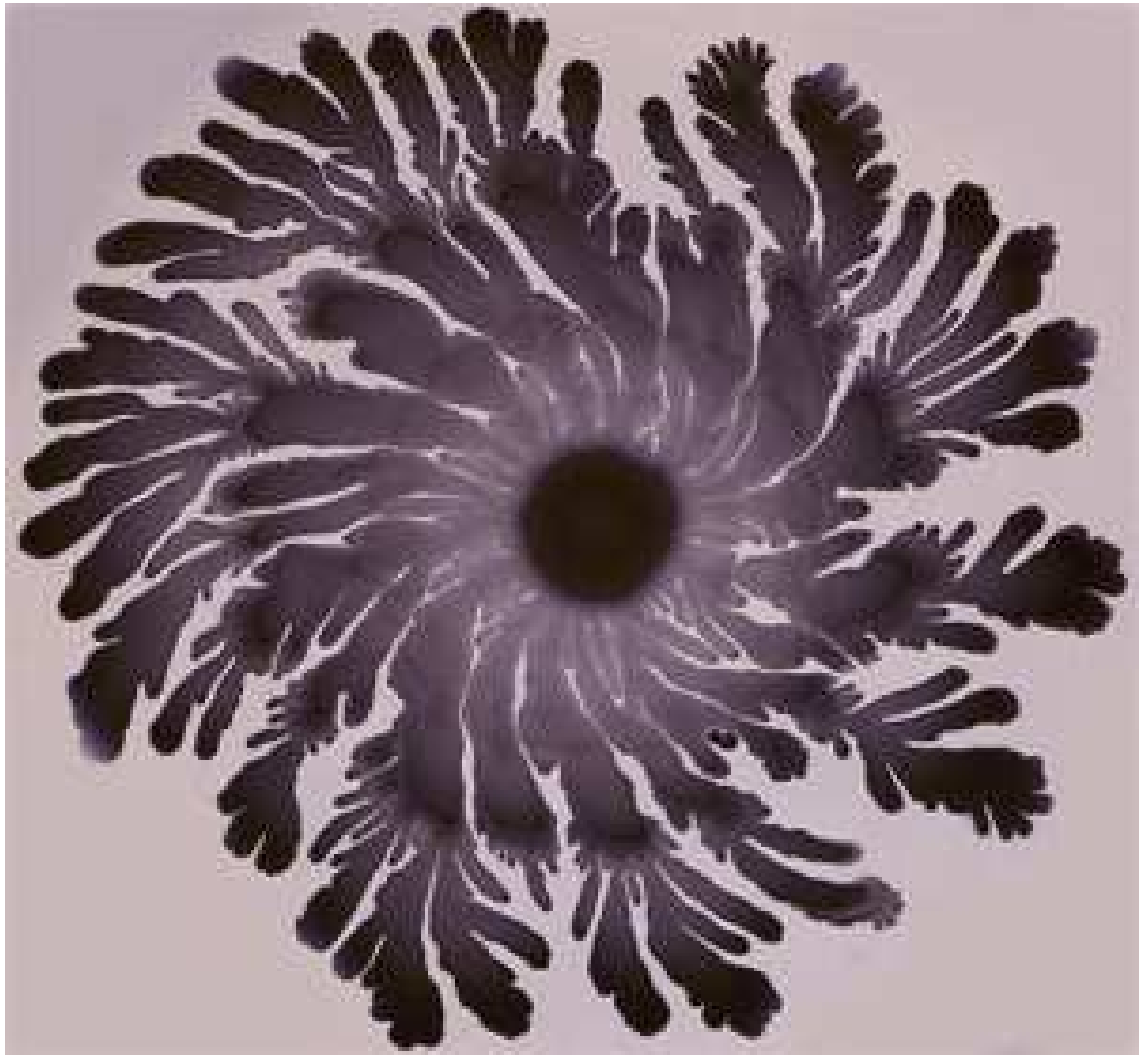


Goethite in agate

Natural Cu

Manganese oxide in chalcedony

2D aggregate growth
(geometrical critical phenomena)



D=1.7

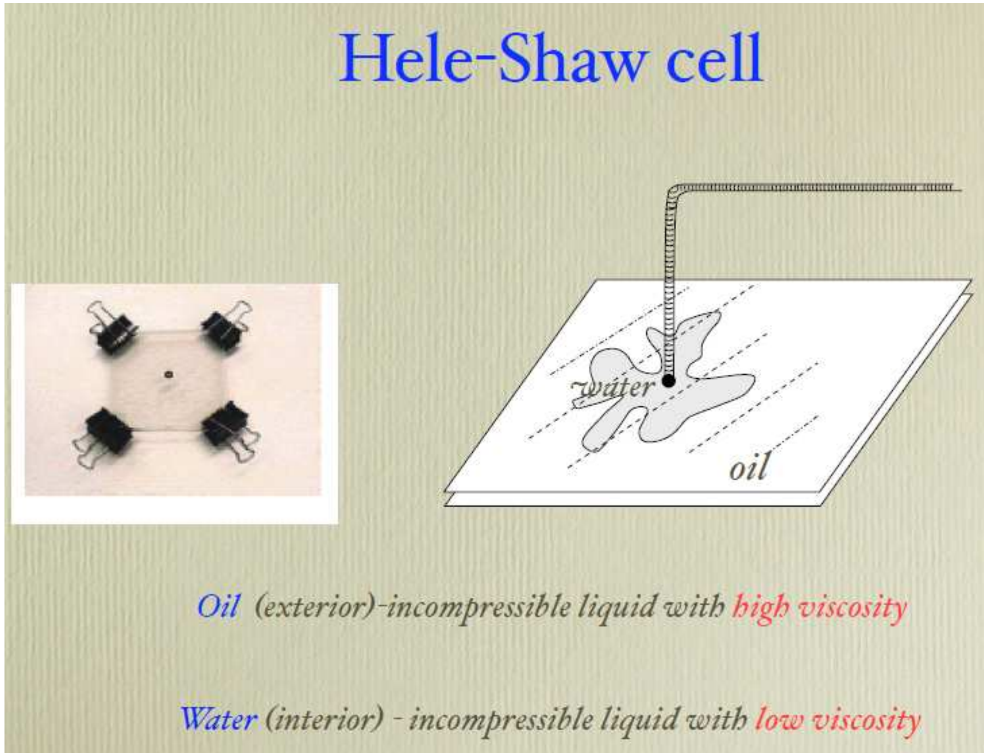
Bacteria colonia Bacillus subtilis
from the site www.igmors.u-psud.fr

2D aggregate growth
(geometrical critical phenomena)

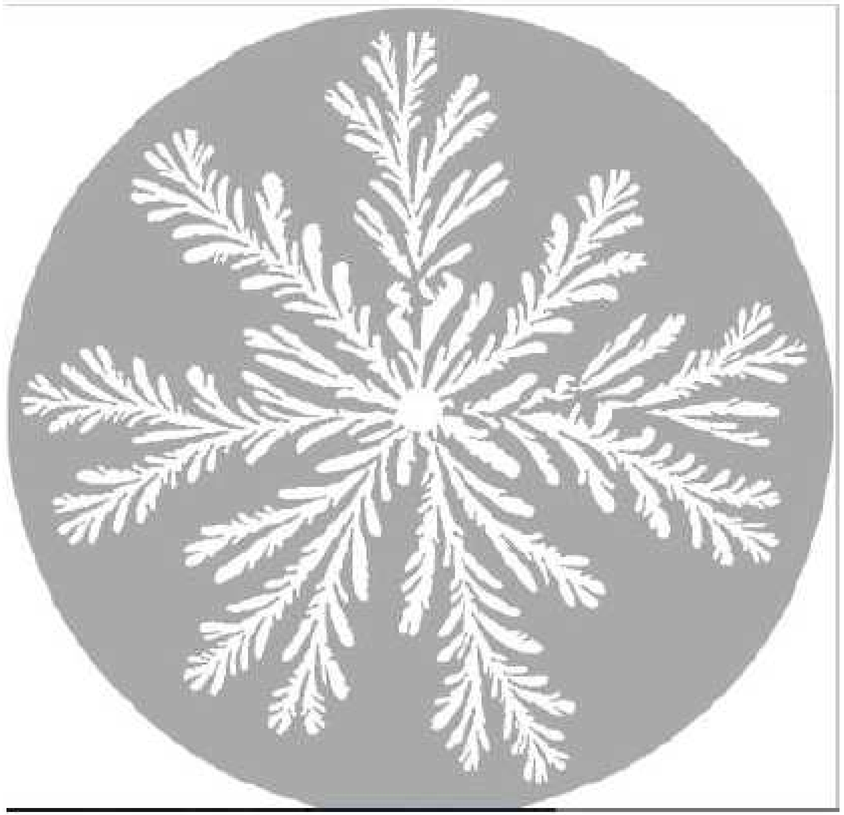
D=1.6-1.9

Nano-size clusters of contamination
on the clean crystal surface

2D aggregate growth
(geometrical critical phenomena)



after Sharon, Moore, McCormick, and Swinney, University of Texas at Austin



D=1.7

2D aggregate growth
(geometrical critical phenomena)

Models:

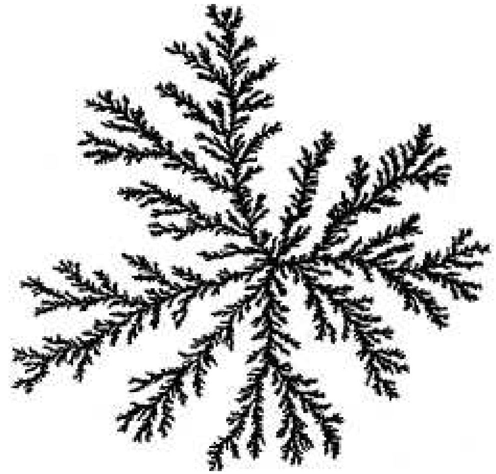
- Diffusion limited aggregation - DLA
- Dielectric breakdown model - DBM
- Laplacian growth
- Iterative conformal maps (Hastings-Levitov dynamics)

2D aggregate growth (geometrical critical phenomena)

Diffusion limited aggregation – DLA
Witten and Sander, PRL, 1981

1. Place seed at origin (0,0), N=1
2. Particle starts at radius of birth Rbirth
3. Diffusion in space
4. If touch, it sticks, N=N+1
5. If particles goes out of the radius of death R_{death} it is killed
6. New iteration – from step 2.

D=1.66-1.7
on the square lattice



D=1.72
Off-lattice

2D aggregate growth
(geometrical critical phenomena)

Dielectric Breadkdown Model – DBM

Niemeyer, Pietronero and Wiesmann, PRL, 1984

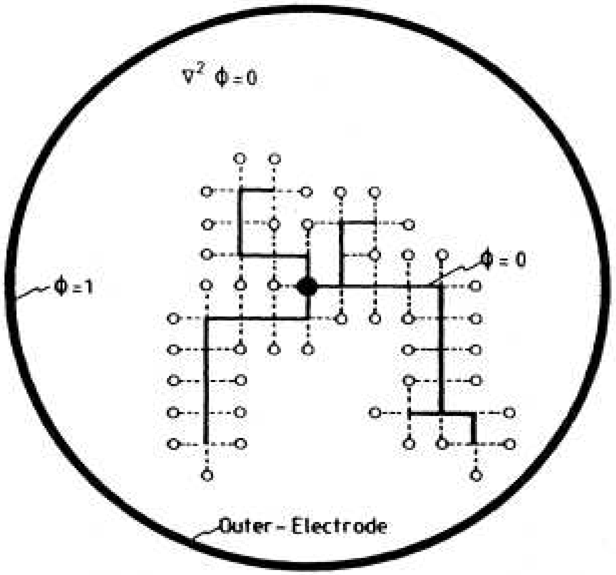


FIG. 2. Iterative mathematical nature of the DBM. The growth process corresponds to an irreversible dynamical process with long-range correlations in both space and time. No statistical weight can be assigned to a given configuration without taking into account its entire history. In the circle is a schematic of the DBM. The central point represents one of the electrodes ($\phi=0$), while the other electrode is given by a circle at large distance ($\phi=1$). The discharge pattern (black dots and bonds) is equipotential with the central electrode ($\phi=0$). The dashed bonds represent the candidates for the next growth processes, and their relative growth probabilities are proportional to the potential gradient (local field).

2D aggregate growth (geometrical critical phenomena)

Iterative conformal maps

Hastings and Levitov, Phyisa D,1998

D=1.65 - 1.72

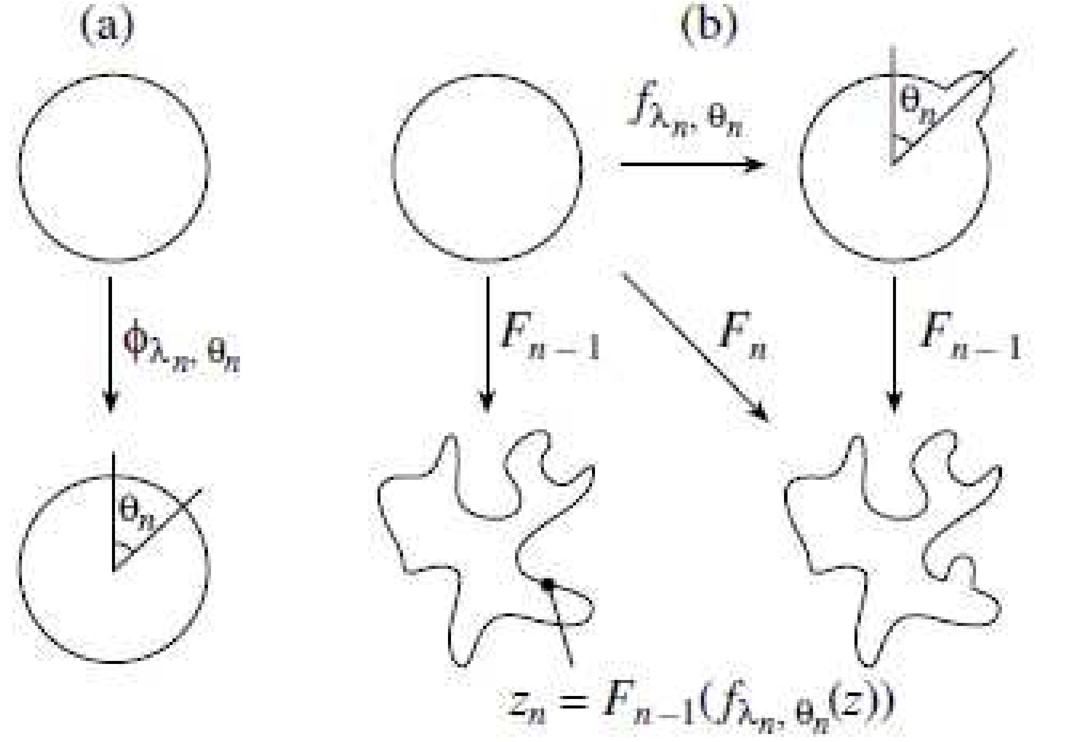


Fig. 1. Action of the mappings $\phi_{\lambda_n, \theta_n}$, f_{λ_n, θ_n}, and F_{n-1}, F_n.

2D aggregate growth (geometrical critical phenomena)

Hele-Shaw dynamics

Wiegmann, et al., arXiv:0811.0635

D=?

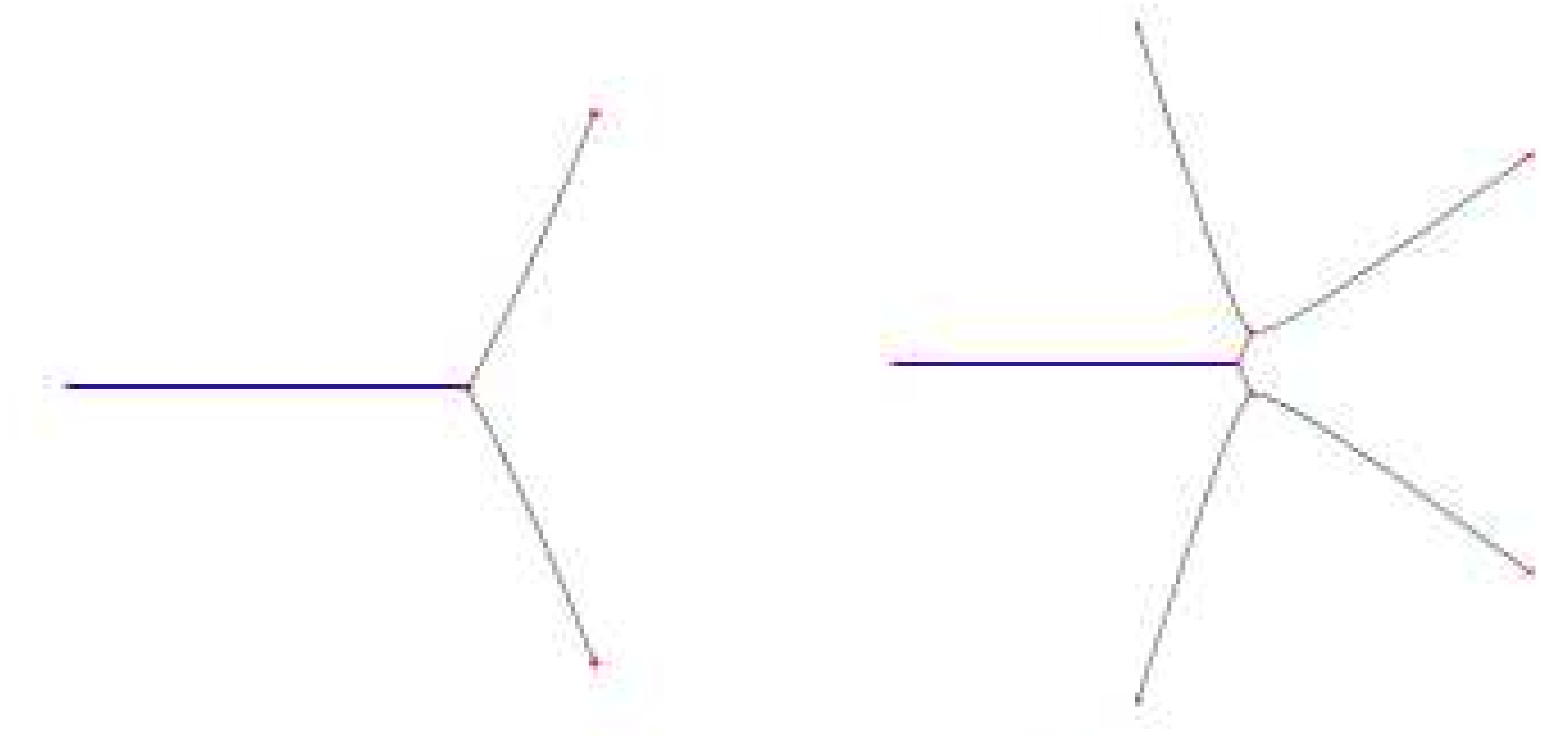


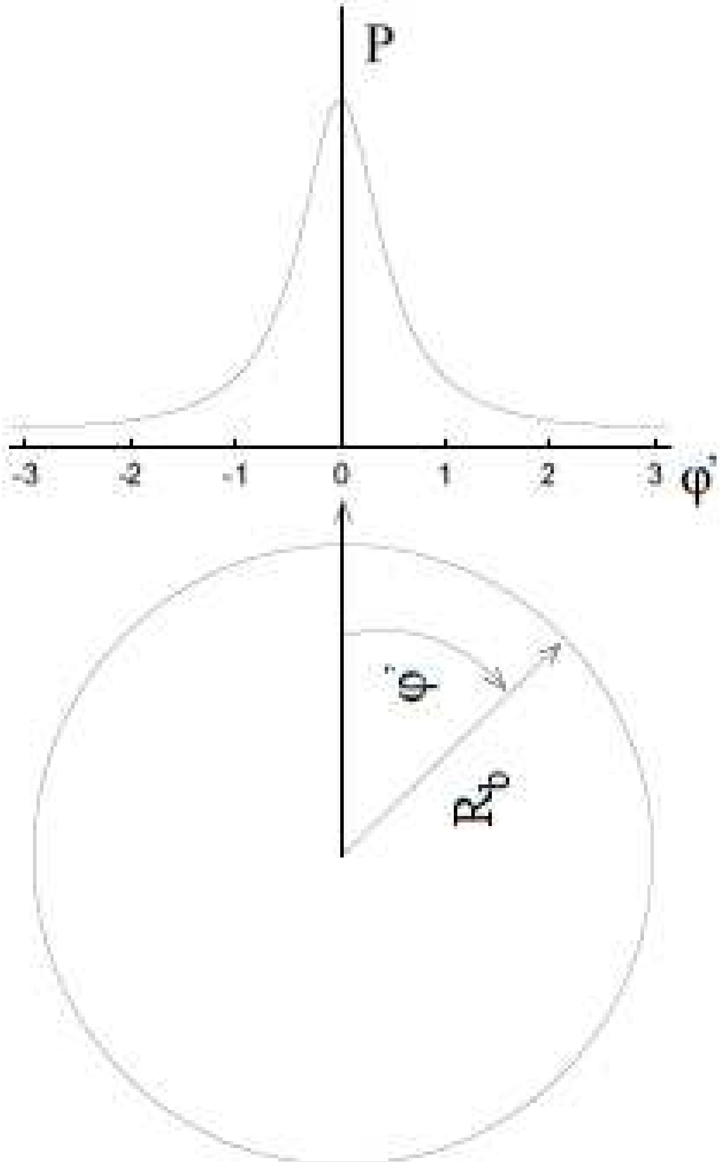
FIG. 2: A growing and branching shock pattern, with one (left) and two (right) generations of branchings. The bold line along the negative x-axis represents a narrow viscous finger (fluid). At this scale, the viscous finger is vanishingly narrow.

Off-lattice killing-*free* algorithm for DLA model

1. Place seed at origin (0,0), N=1
2. Particle starts at radius of birth Rbirth
3. Diffusion in space
4. If touch, it sticks, N=N+1
5. If particles goes out of the radius of death R_{death} it is returned on R_{birth} with probability

$$P(\varphi) = \frac{1}{2\pi} \frac{x^2 - 1}{x^2 - 2x\cos\varphi + 1}$$

6. New iteration – from step 2.



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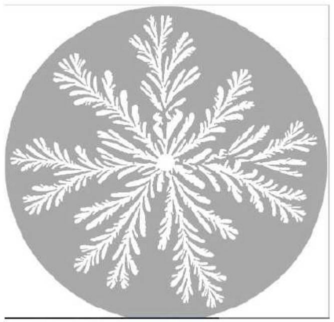
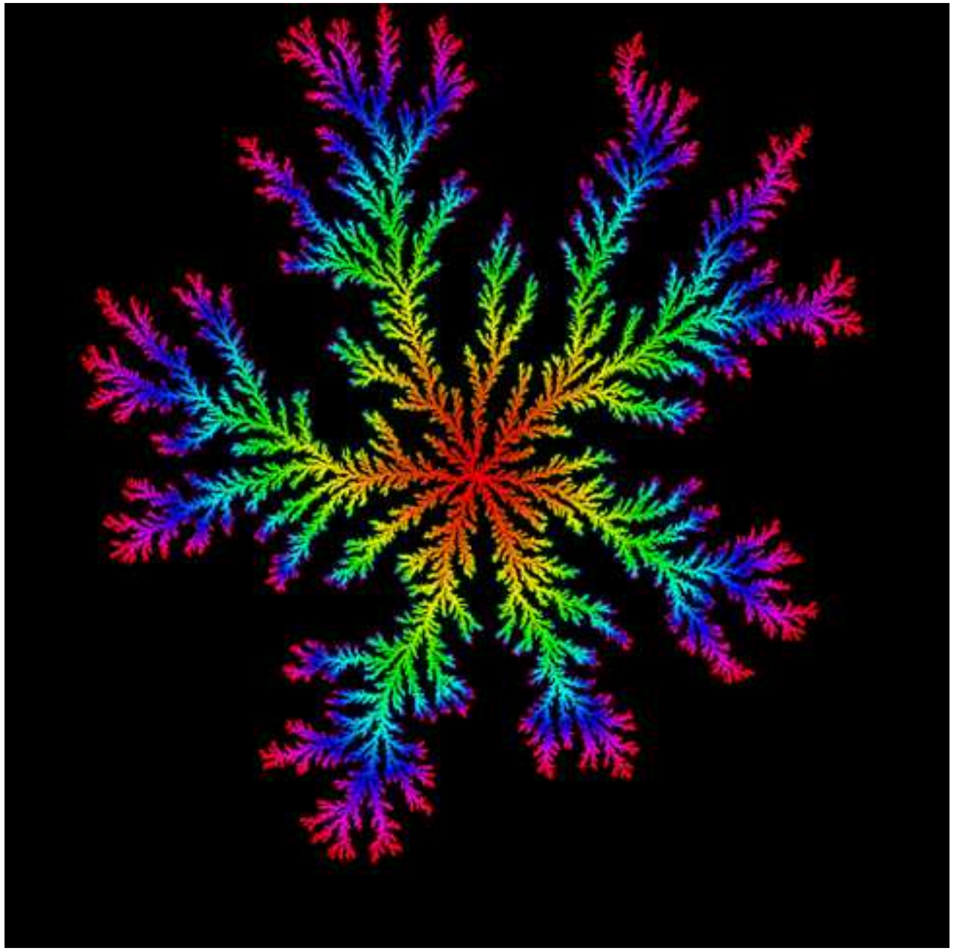




50 000 000 particles

1000 clusters in each ensemble

Measurement of fractal dimension

(harmonic measure)

$dq = P(l)dl$, where $P(l)$ is the probability to stick cluster surface at the point l.

Deposition radius $$R_{dep} = \langle \int r \, dq \rangle$$

Mean-square radius $$R_2 = \langle \sqrt{\int r^2 \, dq} \rangle$$

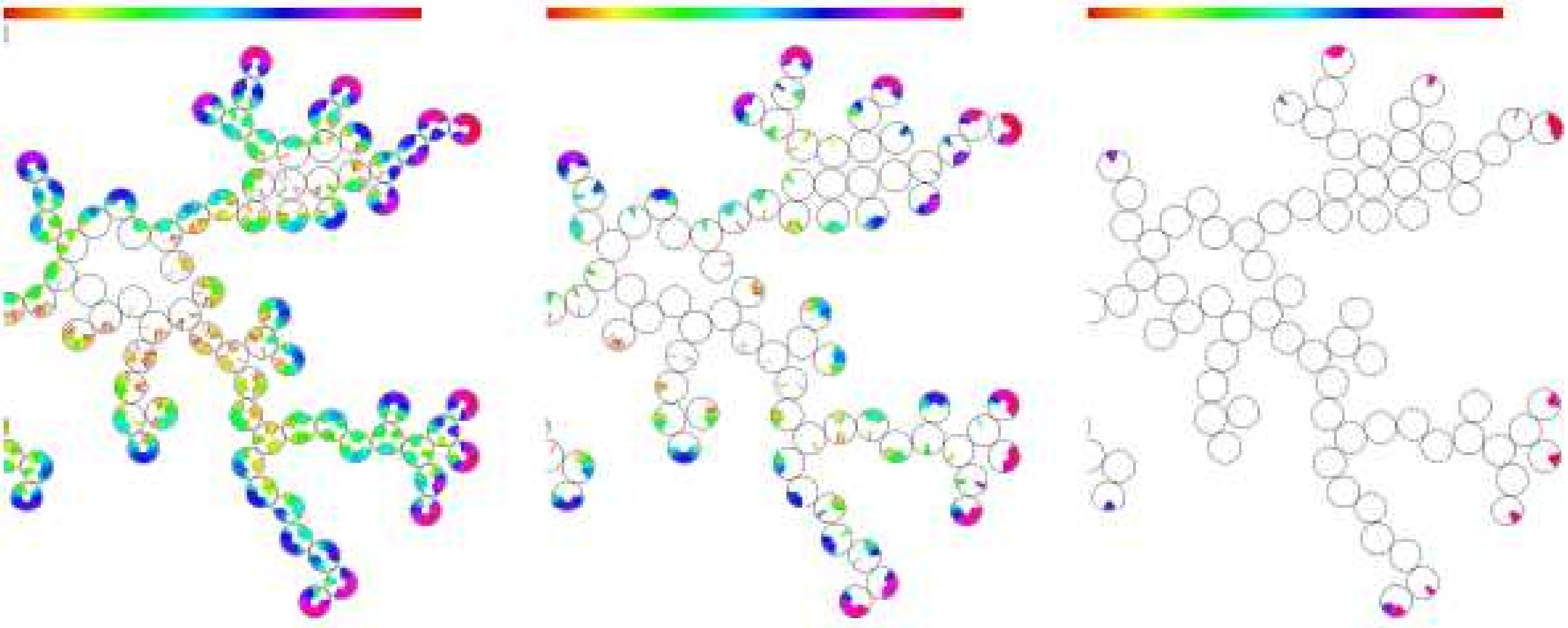
Effective radius $$R_{eff} = \langle \exp \int \ln r \, dq \rangle$$

Maximal radius $$R_{max} = \langle \max_q r \rangle$$

Penetration depth $$\xi = \sqrt{R_2^2 - R_{dep}^2}$$

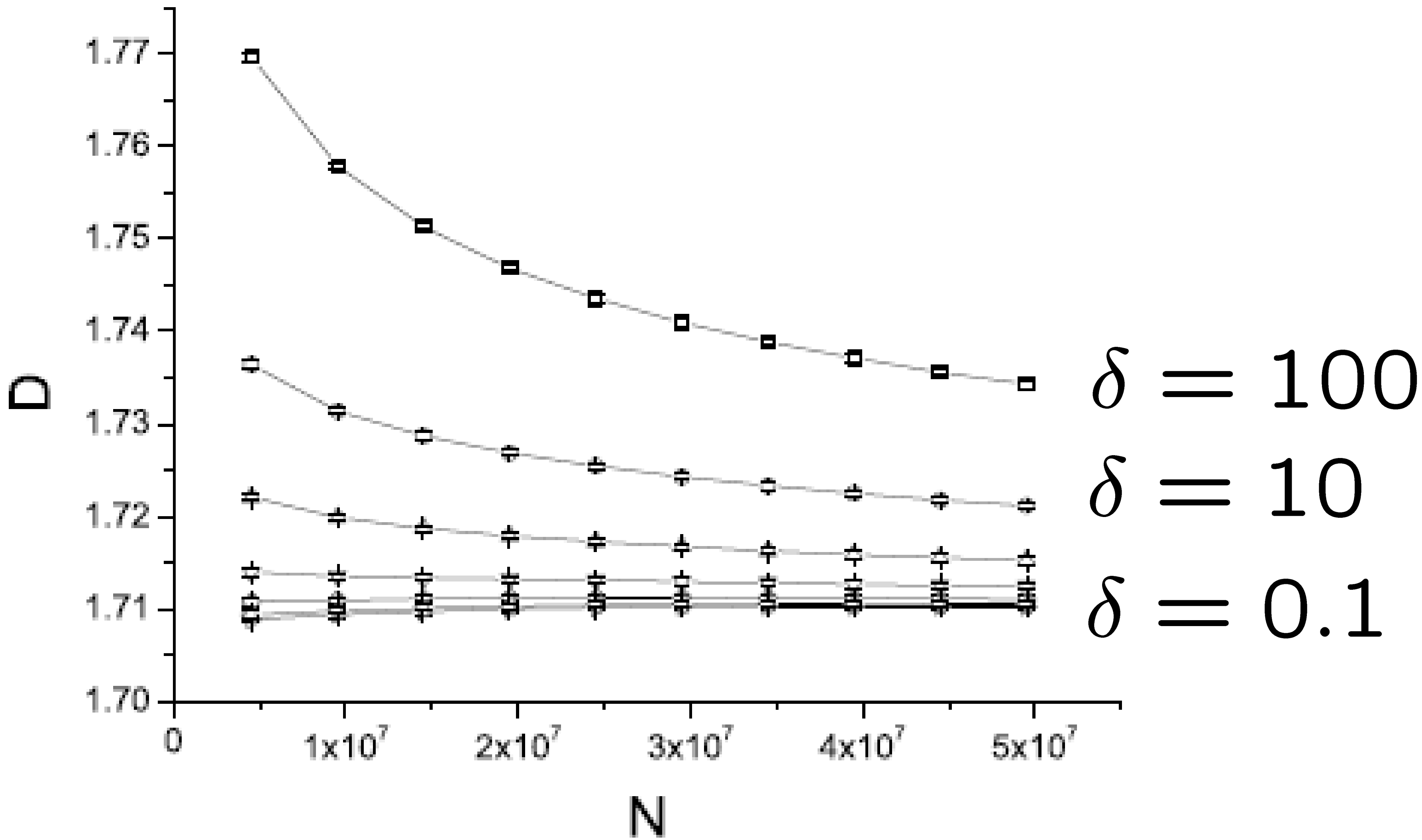
…

Probing harmonic measure with particles of size δ

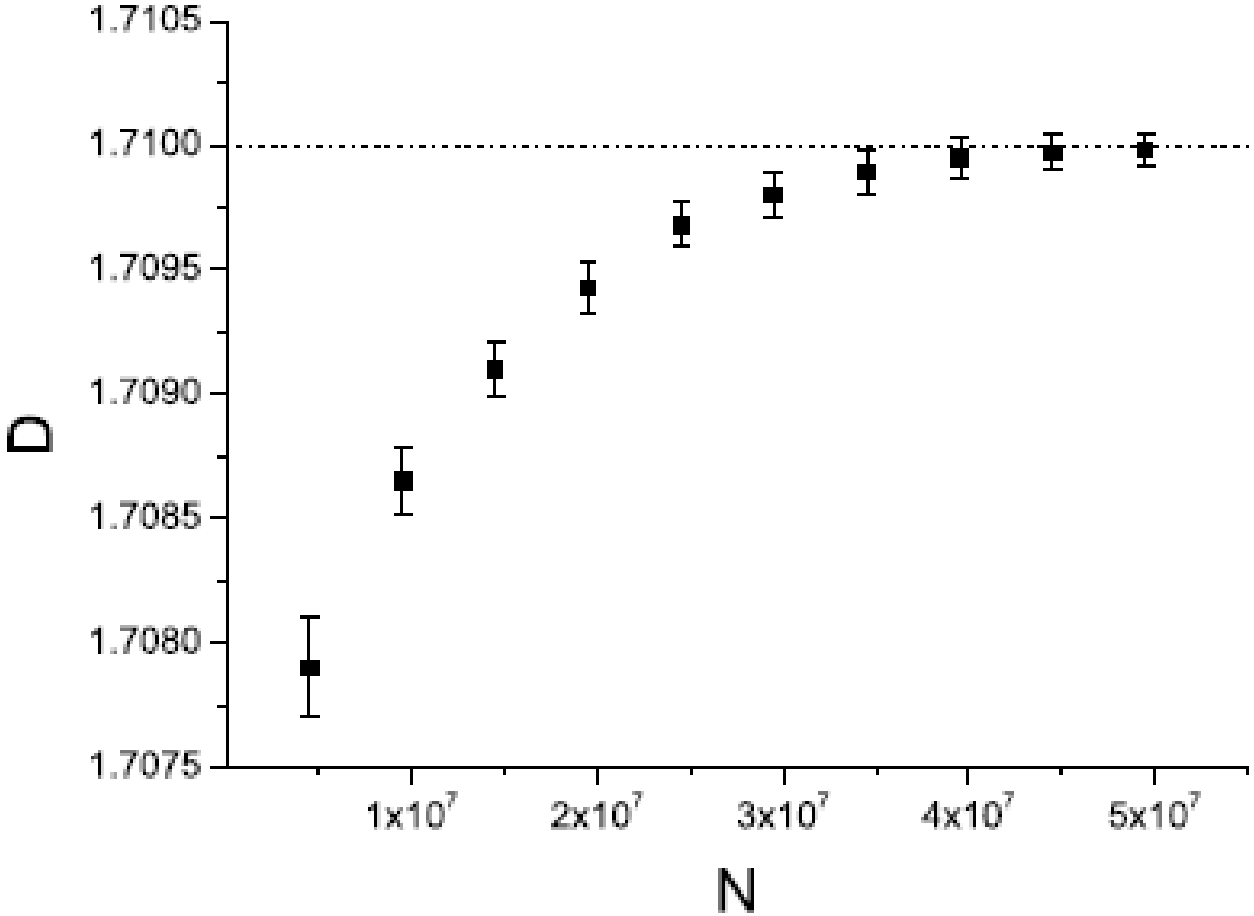


$\delta = 0.1$ $\delta = 1$ $\delta = 10$

Effective fractal dimension



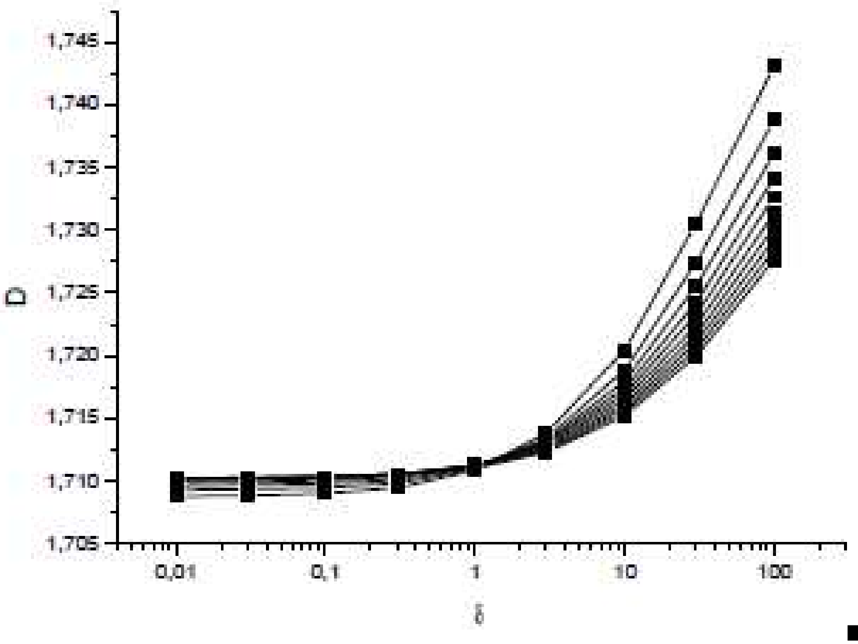
Effective fractal dimension

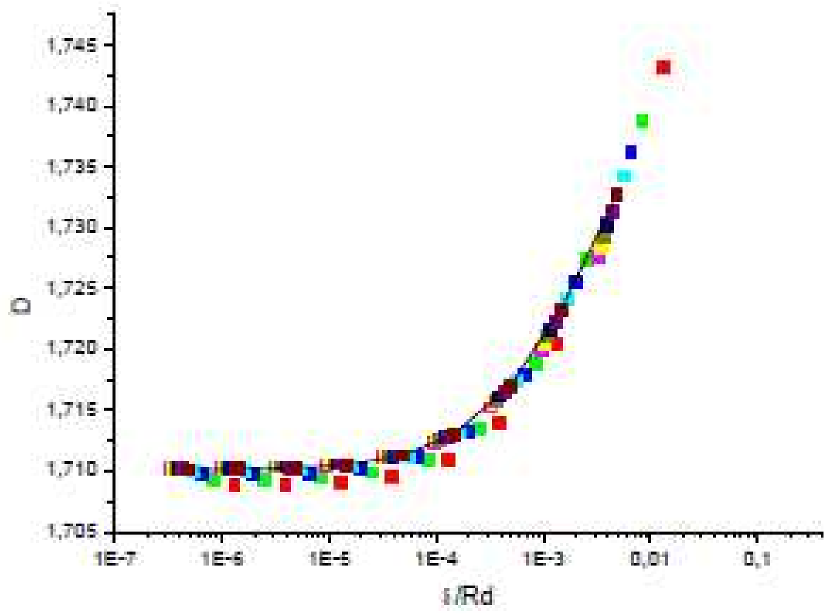


Fractal dimension $D = 1.7100(2)$

Effective fractal dimension

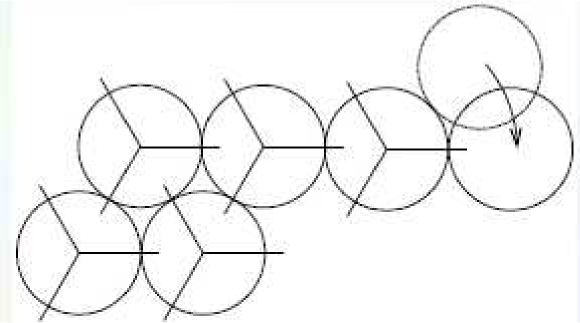
Collapse of the effective D

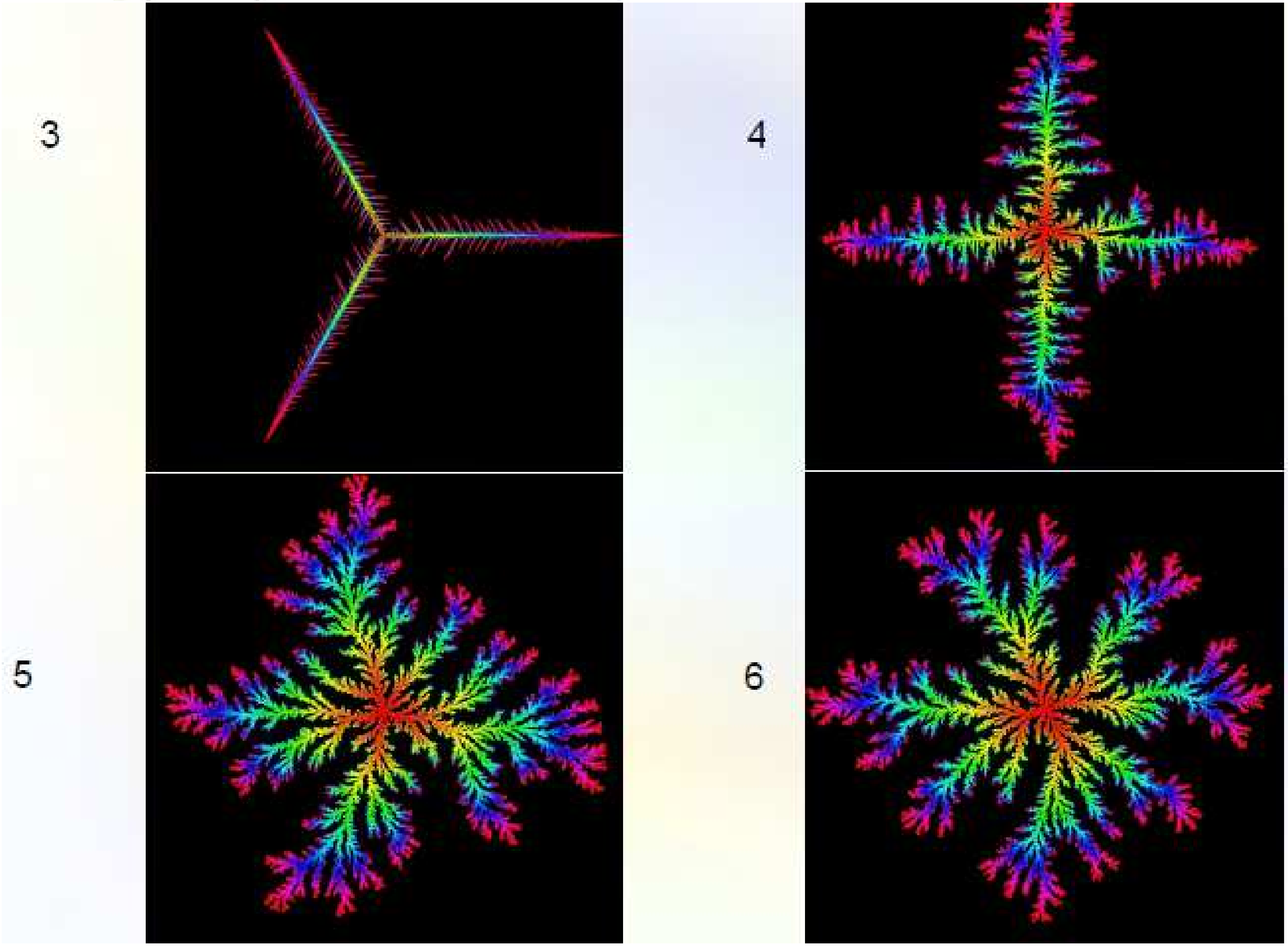




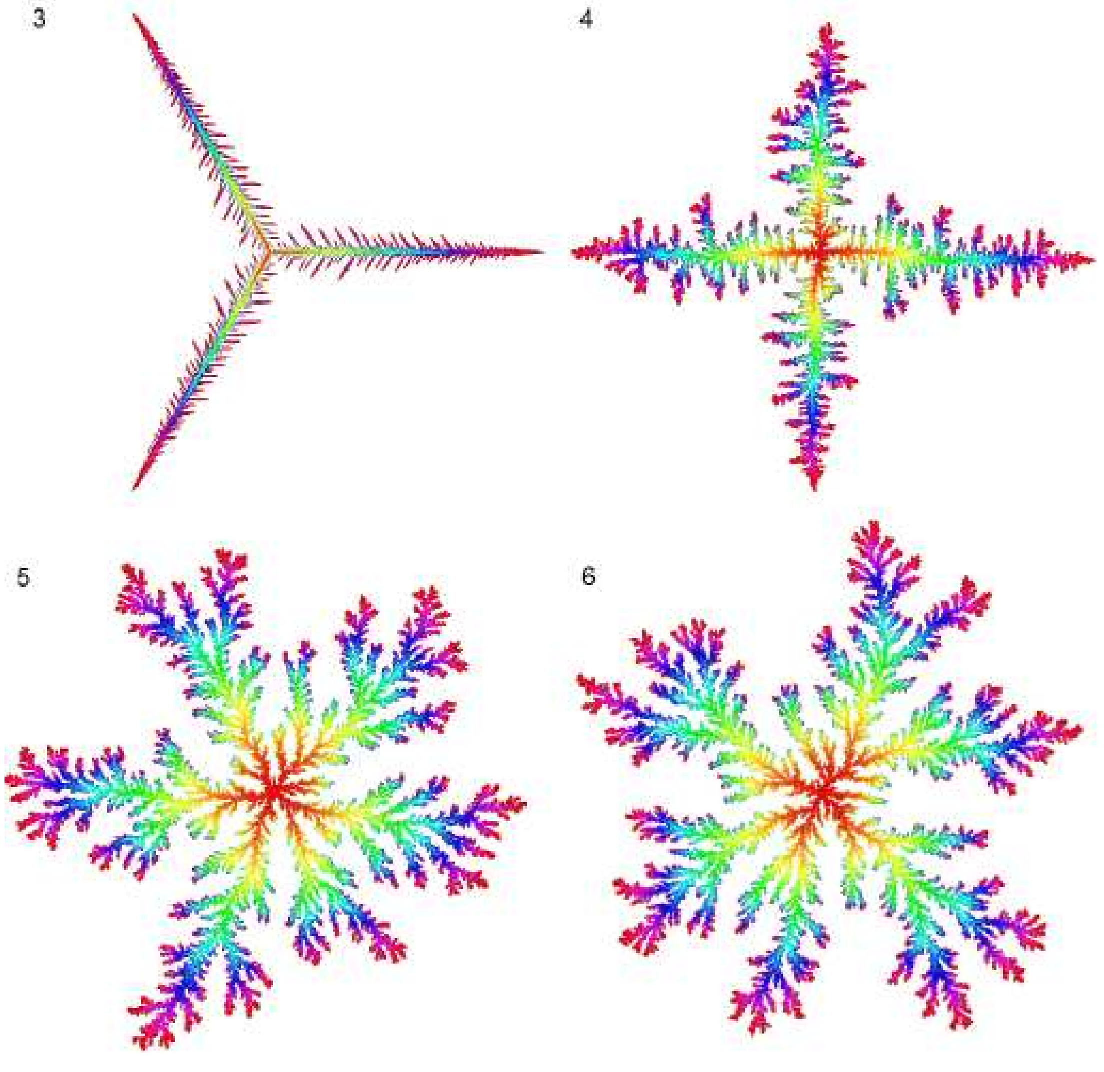
$$D(N,\delta) = D(\delta / R_{dep})$$

Anisotropic clusters



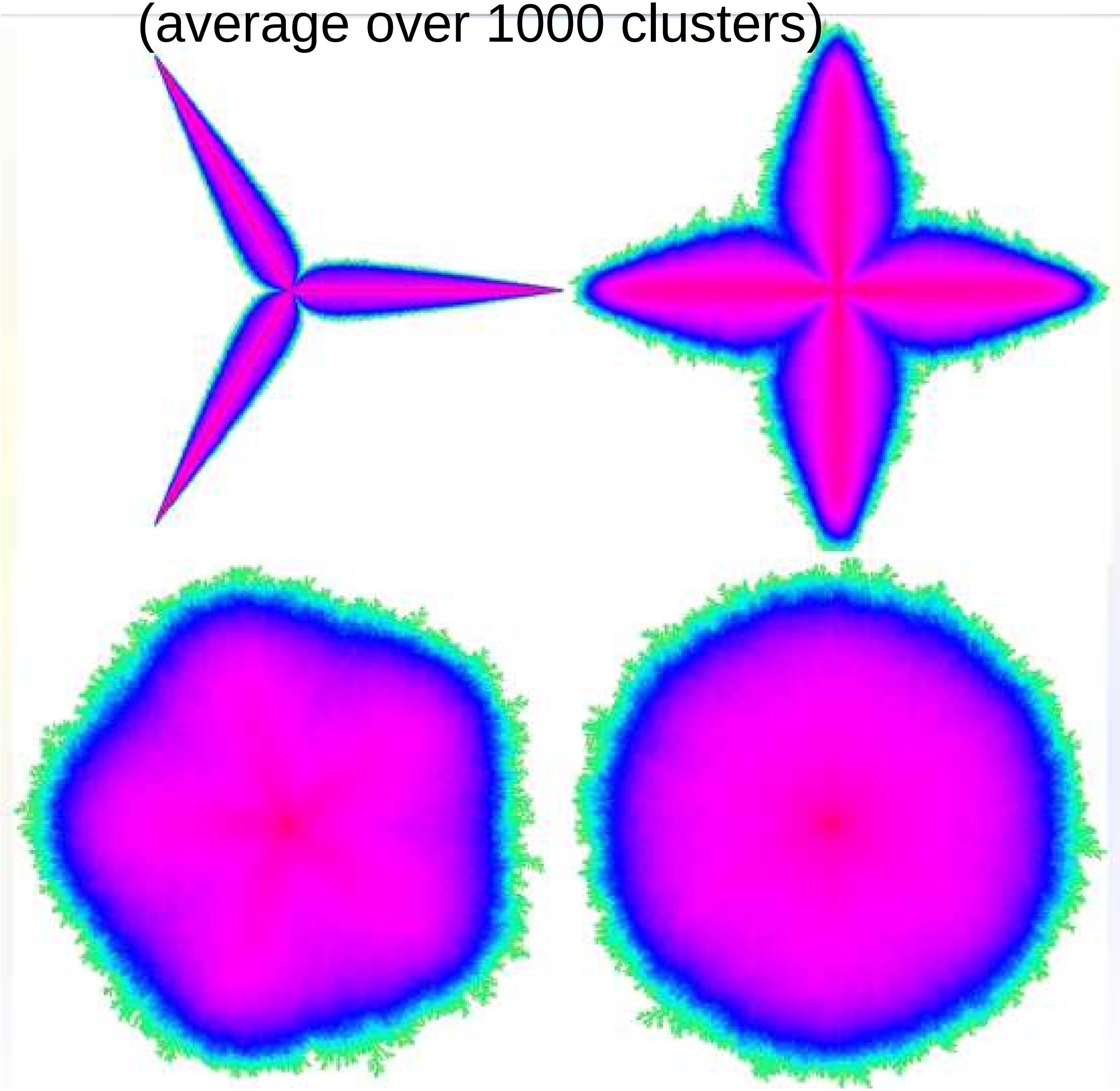


Anisotropic clusters



Density of the particles

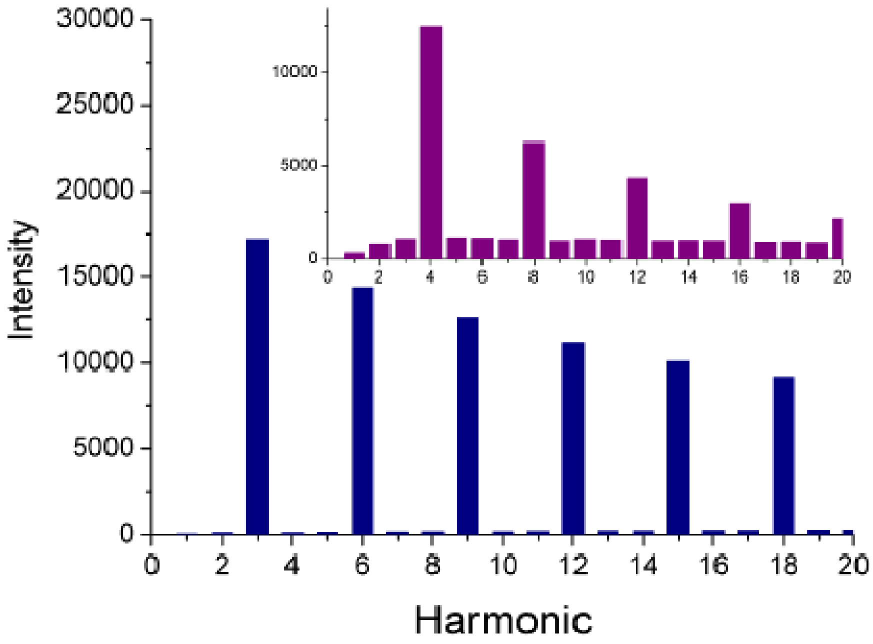
(average over 1000 clusters)



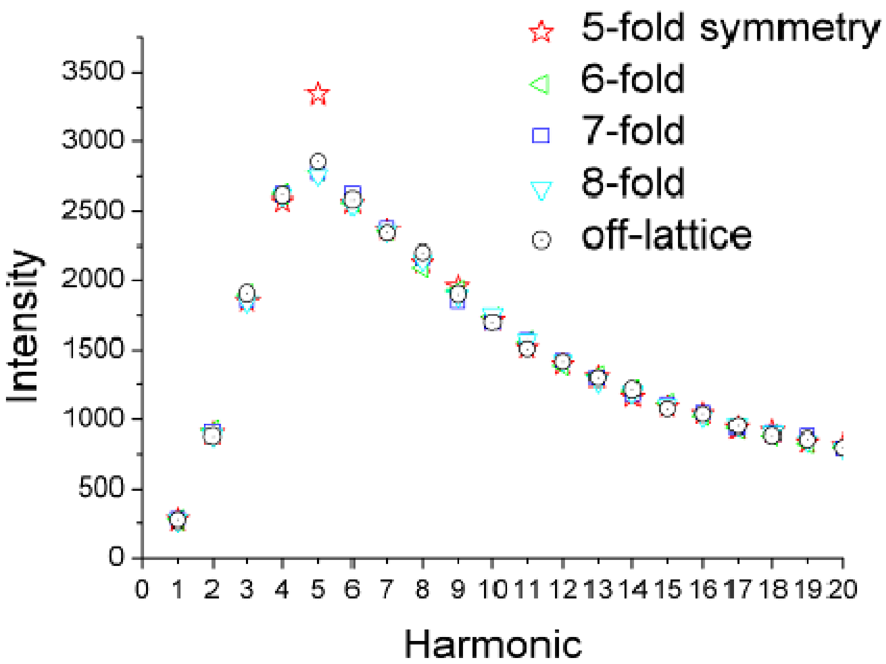
Fourier analysis of the spectrum

$$P(\phi) = a_0 + \sum a_k \sin(kx) + b_k \cos(kx)$$

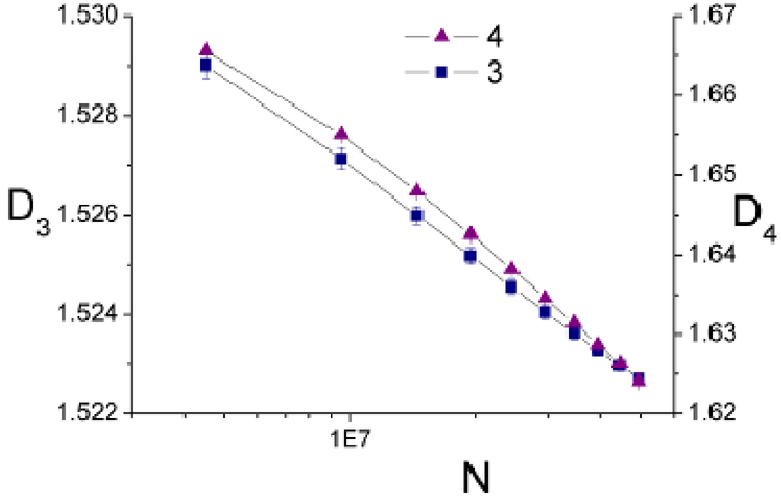
$$I_k = \sqrt{a_k^2 + b_k^2}$$

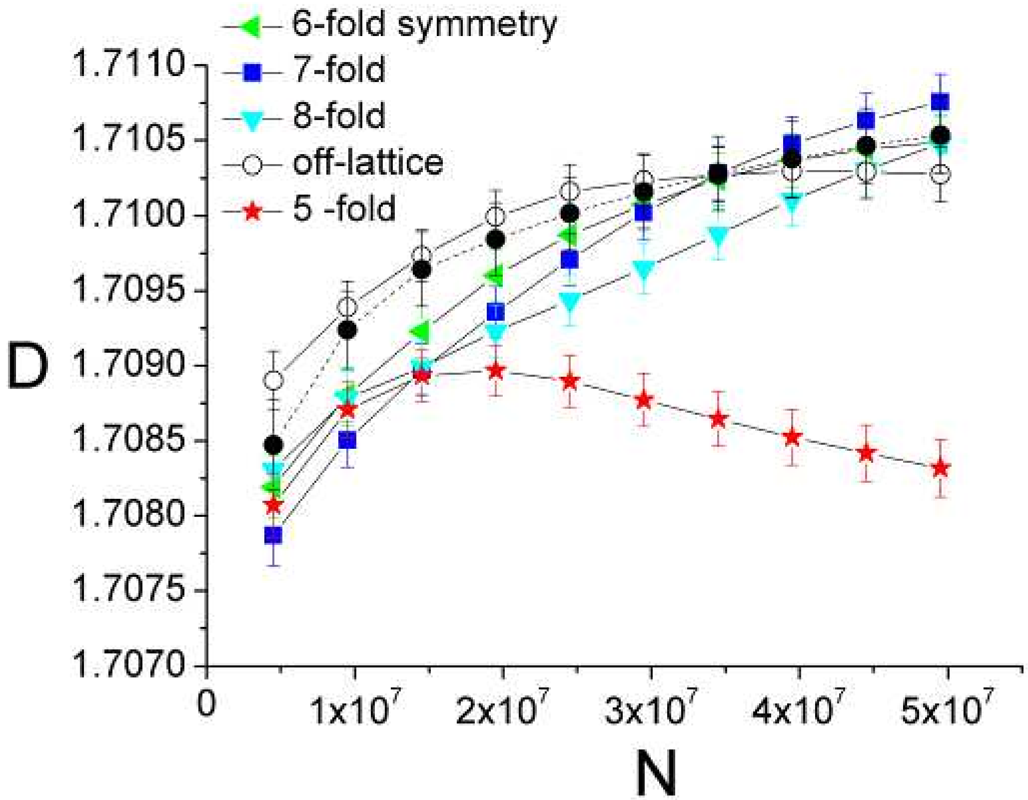


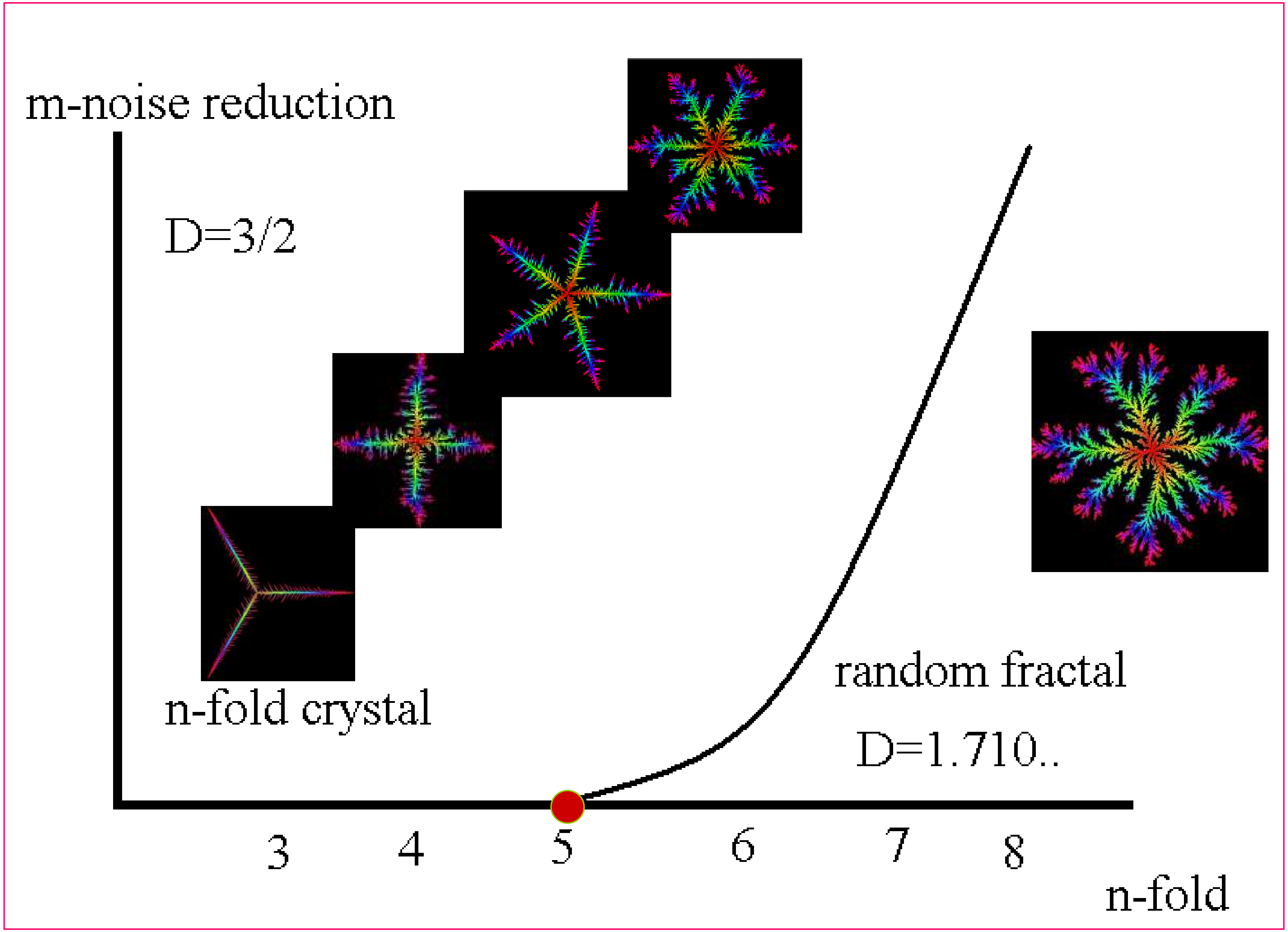
4-fold symmetry
3-fold symmetry



Fractal dimension estimation






m-noise reduction
D=3/2
n-fold crystal
random fractal
D=1.710..
3
4
5
6
7
8
n-fold

Summary

In two dimensional DLA growth (asymptotically) there are only two regimes of growth:

1. n-fold fractal crystal, $D=3/2$
2. random crystal, $D=1.710\ldots$

- Dynamical “phase transition” in DLA model.
- Critical line in the (n-m) plane (n-fold symmetry and m-noise reduction) - orientational transition